Homework Assignment #3
Due: Friday, October 23, 2020 at 5:00 p.m.

1. Consider the BUSY-STUDENT problem. At the beginning of a term, Koidu is given all of the n homework assignments that she will have to complete during the term in all of her courses. For each assignment A_i, she knows the date d_i it is due and the weight w_i of the assignment for her final grade in the course. Assume d_i is a positive integer saying which day of the term the assignment is due. Assume assignments are always due at the end of the specified day. Assume w_i is a positive integer; for example, if the assignment A_i is worth 5%, $w_i = 5$. She also knows it will take her ℓ_i days of work to complete the assignment. (Assume ℓ_i is a positive integer, because once Koidu finishes working on one homework assignment, she does not like to start another one until the beginning of the next day.) Koidu is an extremely good student, so if she completes an assignment (on time), she will get a grade of 100% on it. If an assignment is not completed on time, Koidu gets 0% for it. The problem for Koidu is to figure out whether it is possible to schedule her work on assignments so that the total weight of the assignments she completes on time is at least W, where W is an given positive integer.

For example, consider the following input for the BUSY-STUDENT problem: $n = 4$, $W = 6$ and the assignments have the following attributes:

i	ℓ_i	d_i	w_i
1	2	2	2
2	2	4	2
3	4	4	5
4	1	1	4

There are three schedules that will complete assignments on time:

- First do assignment A_1, which takes 2 days and is due at the end of day 2 of the term. Then do assignment A_2 which takes two more days, so can be completed by the end of day 4 and is due at the end of day 4 of the term. This schedule has total weight 4.
- First do assignment A_4, which takes 1 day and is due at the end of day 1 of the term. Then do assignment A_2 which takes two more days, so can be completed by the end of day 3 and is due at the end of day 4 of the term. Rest on the fourth day of the term. This schedule has total weight 6.
- Do assignment A_3, which takes 4 days and is due at the end of day 4 of the term. This schedule has total weight 5.

(There is no better schedule because at most one of A_1 and A_4 can be completed on time, and if A_3 is completed on time, no other assignment can be done on time.)

The optimal schedule for this input is to do A_4 and then A_2. This schedule shows that it is possible to achieve the total weight $W = 6$, so the answer for this input is yes.

Note that the size of the input is the total length of a string used to represent all of the inputs (using binary notation for all of the integers).

(a) Many job scheduling problems can be solved efficiently using a greedy algorithm. Such an algorithm builds an optimal schedule by first sorting the jobs into some order, and then going through the jobs one-by-one, choosing to include or exclude each job based on some simple criterion (*without* looking ahead at the later jobs in the sorted order).

Suggest a reasonable greedy approach that you might try to solve the BUSY-STUDENT problem that would run in time polynomial in n and $\log W$, if you assume that you can do arithmetic and comparisons between two integers in $O(1)$ time.

(b) Give an input that would cause your algorithm in part (a) to give the wrong answer.

(c) Show that BUSY-STUDENT is in NP.

(d) Show that SUBSET-SUM $\leq_p$ BUSY-STUDENT.