

Review Questions

1. Prove that $\neg(\neg p \Leftrightarrow (r \vee p))$ is logically equivalent to $r \Rightarrow p$.
2. Use a proof by contradiction to show that if n is an integer and n^2 is even, then n is even.
3. Is the following statement true or false?
 $\forall x \text{ in } \mathbb{R}, \exists y \text{ in } \mathbb{R} \text{ such that } y \geq 0 \wedge (y = x \vee y = -x)$.
 Explain why your answer is correct.
4. Is the following statement true or false?
 For all sets A, B and C , $A - (B \cup C) \subseteq (A - B) \cap (A - C)$.
 Prove your answer is correct.
5. Let $f : B \rightarrow C$ and $g : A \rightarrow B$. Prove that if $f \circ g$ is onto then f is onto.
6. Prove that $x^3 - 10x^2$ is *not* $O(x^2)$.
7. Prove that for every positive integer n , $\sum_{k=1}^n k2^k = (n-1)2^{n+1} + 2$.
8. Yark University has 40000 students. Each student takes 5 classes each term. The University offers 1000 classes each term. The largest classroom at Yark holds 180 students. Is this a problem? Explain why.
9. Consider the domain of all people.
 Let $P(x, y)$ represent the statement “ x is the parent of y ”.
 - (a) Translate the following formulas into clear and precise English.
 $\exists x \forall y P(x, y)$
 $\forall y \exists x P(x, y)$
 - (b) Express the statement “Somebody has no grandchildren” using only the predicate P .
10. Prove that for all integers a, b and c , the product of some pair of the three integers is non-negative. (In other words, show that $ab \geq 0$ or $ac \geq 0$ or $bc \geq 0$.)
11. Let $A = \{0, 1, 2\}$ and $B = \{1, 3\}$. List the elements of each of the following sets.
 - (a) $A - B =$
 - (b) $A \times B =$
12. Let $f : A \rightarrow B$. Let S and T be subsets of A .
 - (a) Prove that $f(S \cap T) \subseteq f(S) \cap f(T)$.

(b) Give an example of a function f and sets S and T such that $f(S \cap T) \neq f(S) \cap f(T)$. Briefly explain why your answer is correct.

13. Give a good lower bound on $\sum_{i=1}^n i^{1.5} (\log_2 i)^2$ using Ω notation. Prove your answer is correct.

14. Use the integral method to get a good upper bound on $\sum_{i=0}^n i e^{2i^2}$.

15. Is $\Omega(n) \subseteq \Omega(n + \sqrt{n})$? Prove your answer is correct.

16. Is $3^{\log n} \in O(2^{\log n})$? Prove your answer is correct.

17. Beside each function $f(n)$ in the left column, write down the number of the **first** function $g(n)$ in the right column such that $f(n) \in O(g(n))$.

(a) $n^{1.9} (\log_2 n)^2$ _____

1. 1

(b) $\sum_{i=1}^n \sqrt{i} \log_2 i$ _____

2. $\log n$

(c) $(n\sqrt{n} + 7n + 2)^2$ _____

3. \sqrt{n}

(d) $10 \cdot 2^{\log_4 n}$ _____

4. $\sqrt{n} \log n$

(e) $\sum_{i=1}^n \frac{n}{i \log n}$ _____

5. n

6. $n \log n$

7. n^2

8. n^3

9. 2^n

10. n^n

18. Prove $n^2 \leq 2^n$ for all natural numbers $n \geq 4$.

19. Let $f: A \rightarrow B$ be a function. For any set $C \subseteq B$, define $f^{-1}(C)$ to be the set $\{a \in A : f(a) \in C\}$. Prove that for every $f: A \rightarrow B$ and subsets S and T of B we have $f^{-1}(S) \cap f^{-1}(T) = f^{-1}(S \cap T)$.

20. Show that if 51 distinct numbers are chosen among $\{1, 2, 3, \dots, 100\}$ then there must be two numbers among them whose sum is 101.