Homework Assignment #6
Due: March 2, 2020 at 2:30 p.m.

1. Suppose you are planning a bicycle trip from Toronto to Vancouver this summer. You have chosen your route. You have a list of the n hotels along the route. For $i = 1..n$, let d_i be the distance from Toronto to the ith hotel along the route. (Here, each d_i is a floating point number and you can assume the hotels are sorted so that $d_1 < d_2 < d_3 < \cdots < d_n$.) Assume the last hotel is at your destination in Vancouver. Your goal is to travel as close to 100 km per day as possible and then sleep in a hotel each night. To formalize this problem, you define a penalty of $(x - 100)^2$ for a day on which you travel x km. You wish to plan out your route to minimize the sum of the penalties for all of your days of travelling. Note that you are not specifying in advance how many days it will take you to reach Vancouver.

Use dynamic programming to find the optimal sequence of hotels to stay in.

(a) Define the array that you will use to solve the problem. State the dimensions of the array. Describe, in English, what the value in each entry of the array represents.

(b) Give a recurrence that can be used to compute the entries of your array. Include the base cases.

(c) Provide pseudocode for computing the entries of the array efficiently. In your pseudocode, you can store additional information that may be needed to solve part (e), below.

(d) What is the running time of the pseudocode in part (c)? Use Θ notation to state your answer in terms of n. Assume that all distances and indices can be stored in a single word of memory.

(e) After the computation in (c) has been done, you might want to print out an optimal itinerary that achieves the minimal total penalty. Give pseudocode for printing this optimal itinerary.

2. Consider the same problem as in Question 1, except that now you want to specify that you must complete the trip to Vancouver in exactly k days. The goal of the dynamic programming algorithm developed in this question is to find the optimal achievable penalty under this additional constraint. If there is no route that completes the trip in exactly k days, we shall say the optimal achievable penalty is ∞.

(a) Define the array that you will use to solve the problem. State the dimensions of the array. Describe, in English, what value will be stored in each entry of the array.

(b) Give a recurrence that can be used to compute the entries of your array. Include the base cases.

(c) How much time would it take to fill in the array? Use Θ notation to state your answer in terms of n and k.