Homework Assignment #1
Due: January 17, 2020 at 2:30 p.m.

When you submit your solution to Assignment 1, you should also hand in the declaration on academic honesty available on the course web page. Without this declaration, your assignment will not be marked. Please do not staple the declaration to your solutions.

1. Consider the following algorithm.

```
1   F(x, n)
2       % precondition: n is a positive integer and x is a positive n-bit integer (i.e., 2^{n−1} ≤ x < 2^n)
3       y ← 2^{⌈n/2⌉−1}
4       step ← y
5       loop
6           exit when step = 1
7           step ← step/2
8           if (y + step)^2 ≤ x then
9               y ← y + step
10          end if
11      end loop
12      return y
13  end F
```

(a) Fill in the following table of values computed by the algorithm.

x	n	$F(x,n)$
1	1	
2	2	
3	2	
4	3	
5	3	
8	4	
10	4	
15	4	
16	5	
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(b) What simple function does F compute?

(c) State a loop invariant that is strong enough to prove the algorithm is correct. (In other words, you should be able to use your invariant to do parts (d) and (e) below.)

(d) Prove the invariant in part (c) is indeed a loop invariant.

(e) Use your invariant, together with the exit condition on line 6 to prove that F correctly computes the function you gave in part (b) for every input that satisfies the preconditions.

(f) Give a careful proof that the algorithm terminates for every input that satisfies the preconditions.

(g) Assuming each line of code takes $O(1)$ time, how much time does the algorithm take in the worst case? Express your answer in terms of n using Θ notation.