MERGE SORT (12.1)

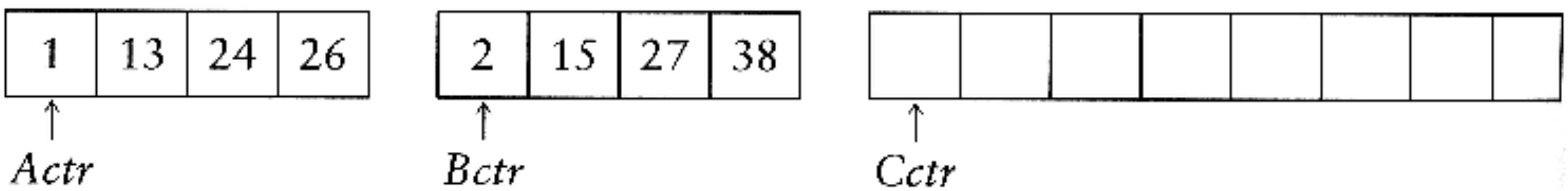
EECS 2011

Goals

- Divide-and-conquer approach
- Recursion
- Solving recurrences
- One more sorting algorithm

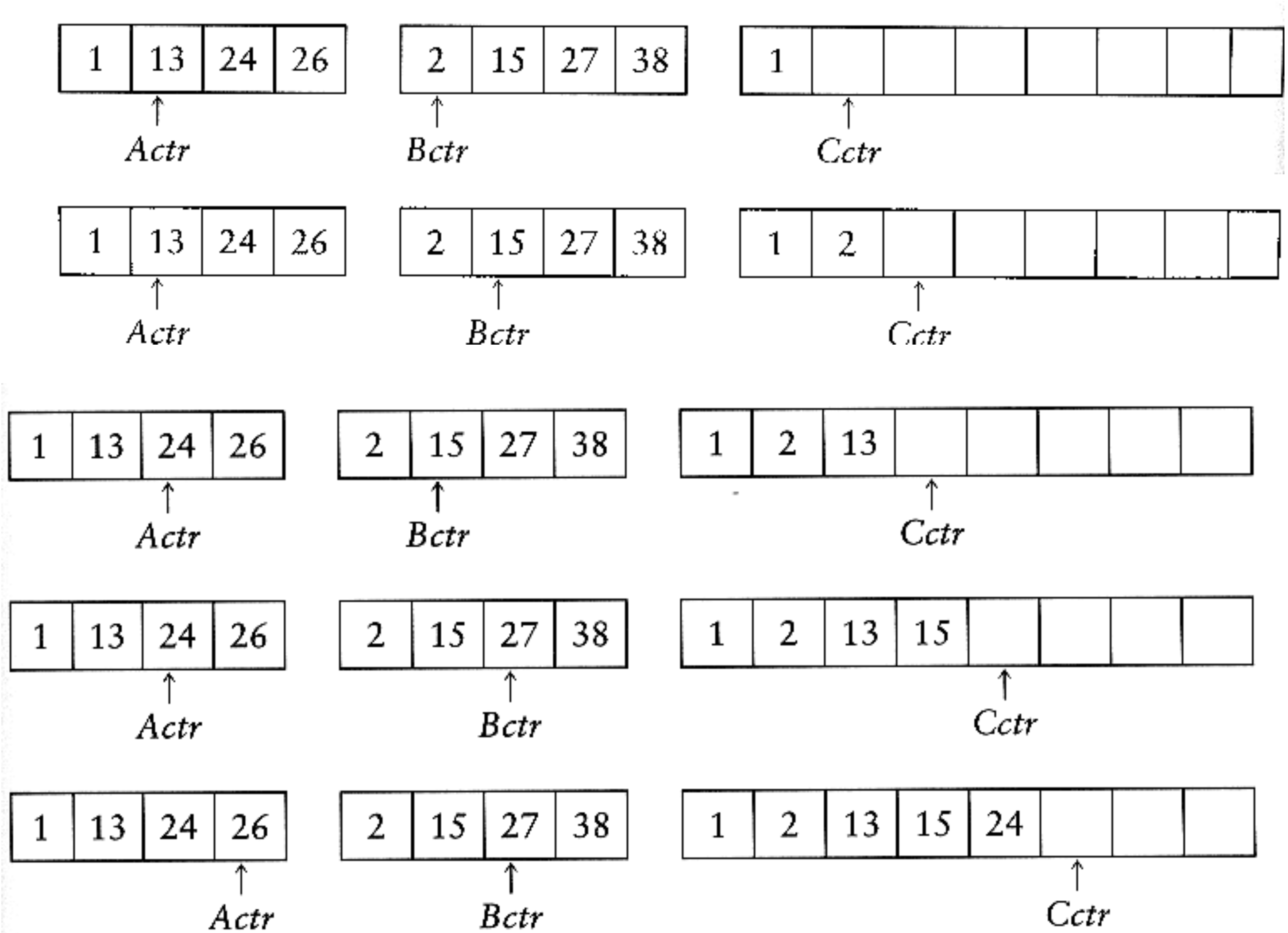
Merging Two Sorted Arrays

- Input: two sorted array A and B
- Output: an output sorted array C
- Three counters: *Actr*, *Bctr*, and *Cctr*
  - initially set to the beginning of their respective arrays

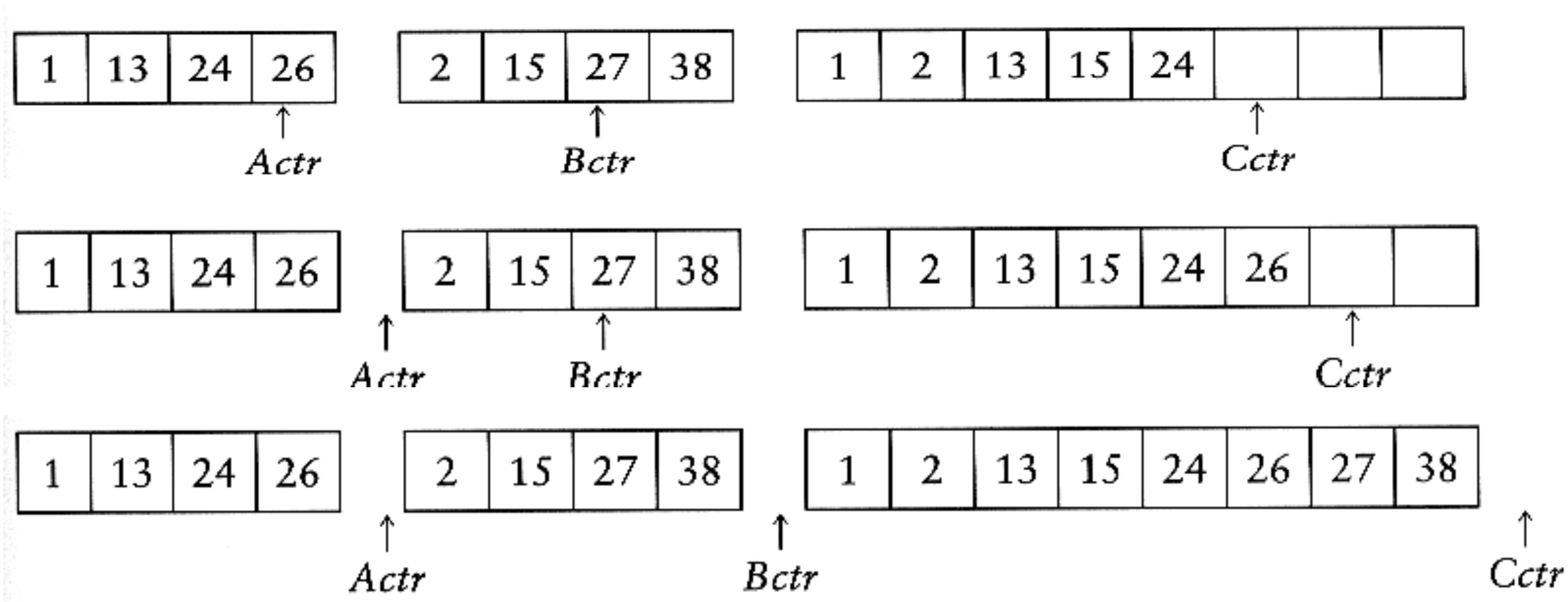


  - The smaller of A[*Actr*] and B[*Bctr*] is copied to the next entry in C, and the appropriate counters are advanced
  - When either input list is exhausted, the remainder of the other list is copied to C.

Merge Two Sorted Arrays: Example



Example: Merge (2)



Merge: Java Code

```
/**
 * Internal method that merges two sorted halves of a subarray.
 * @param a an array of Comparable items.
 * @param tmpArray an array to place the merged result.
 * @param leftPos the left-most index of the subarray.
 * @param rightPos the index of the start of the second half.
 * @param rightEnd the right-most index of the subarray.
 */
private static <AnyType extends
              Comparable<? super AnyType>>
void merge( AnyType [ ] a, AnyType [ ] tmpArray,
int leftPos, int rightPos, int rightEnd )
{
   int leftEnd = rightPos - 1;
   int tmpPos = leftPos;
   int numElements = rightEnd - leftPos + 1;

   // Main loop
   while( leftPos <= leftEnd && rightPos <=
 rightEnd )
      if( a[ leftPos ].compareTo( a[ rightPos ] ) <= 0 )
         tmpArray[ tmpPos++ ] = a[ leftPos++ ];
      else
         tmpArray[ tmpPos++ ] = a[ rightPos++ ];

   while( leftPos <= leftEnd )   // Copy rest of 1st half
      tmpArray[ tmpPos++ ] = a[ leftPos++ ];

   while( rightPos <= rightEnd )
                                   // Copy rest of right half
      tmpArray[ tmpPos++ ] = a[ rightPos++ ];

   // Copy tmpArray back
   for( int i = 0; i < numElements; i++, rightEnd-- )
      a[ rightEnd ] = tmpArray[ rightEnd ];
}
```

Merge: Analysis

- Running time analysis:
  - Merge takes $O(m_1 + m_2)$, where m_1 and m_2 are the sizes of the two sub-arrays.
- Space requirement:
  - merging two sorted lists requires linear extra memory (in merge sort)
  - additional work to copy to the temporary array and back (in merge sort)

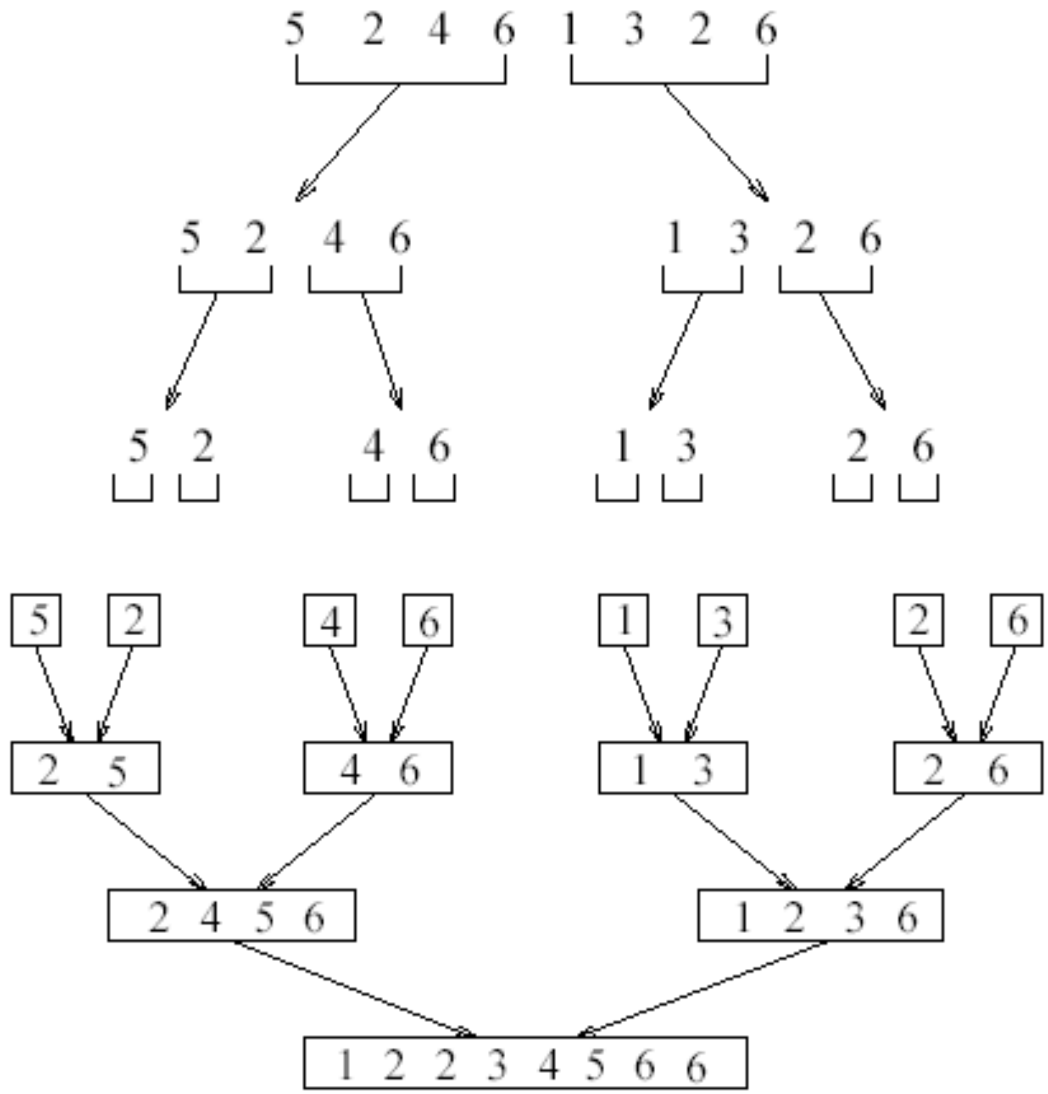
Merge Sort: Main Idea

Based on divide-and-conquer strategy

- Divide the list into two smaller lists of about equal sizes.
- Sort each smaller list *recursively.*
- Merge the two sorted lists to get one sorted list.

Questions:

- How do we divide the list? How much time needed?
- How do we merge the two sorted lists? How much time needed?



Animation: https://www.youtube.com/watch?v=JSceec-wEyw

Merge Sort: Algorithm

- Divide-and-conquer strategy
  - recursively sort the first half and the second half
  - merge the two sorted halves together

```
void mergesort(int & A[], int left, int right)
{
        If (left < right ) {
                int center = (left + right) /2;
                mergesort(A, left, center);
                mergesort(A, center+1, right);
                merge(A, left, center+1, right);
}
```

Dividing

- If the input list is an array A[0..N-1]: dividing takes O(1) time:
  - Represent a sub-array by two integers *left* and *right*.
  - To divide A[*left* .. *right*], compute *center*=(*left*+*right*)/2 and obtain A[*left* .. *center*] and A[*center*+1 .. *right*]
- If the input list is a linked list, dividing takes Θ(N) time:
  - Scan the linked list, stop at the $\lfloor N/2 \rfloor^{th}$ entry and cut the link.

Analysis of Merge Sort

- Let T(N) denote the worst-case running time of ***mergesort*** to sort N numbers.
- Assume that N is a power of 2.

- Divide step: O(1) time
- Conquer step: 2 x T(N/2) time
- Combine step: O(N) time

- Recurrence equation:

$$T(1) = 1$$

$$T(N) = 2T(N/2) + N$$

Solving the Recurrence

$$T(N) = 2T(\frac{N}{2}) + N$$
$$= 2(2T(\frac{N}{4}) + \frac{N}{2}) + N$$
$$= 4T(\frac{N}{4}) + 2N$$
$$= 4(2T(\frac{N}{8}) + \frac{N}{4}) + 2N$$
$$= 8T(\frac{N}{8}) + 3N = \cdots$$
$$= 2^k T(\frac{N}{2^k}) + kN$$

Since $N=2^k$, we have $k=\log_2 n$

$$T(N) = 2^k T(\frac{N}{2^k}) + kN$$
$$= N + N \log N$$
$$= O(N \log N)$$

Next time ...

- Quick Sort (12.2)