HASH TABLES (PART 1)

EECS 2011

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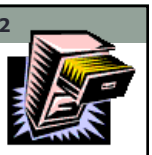
The Map ADT



- get(k): if the map M has an entry with key k, return its associated value; else, return null
- put(k, v): insert entry (k, v) into the map M; if key k is not already in M, then return null; else, return old value associated with k
- remove(k): if the map M has an entry with key k, remove it from M and return its associated value; else, return null
- size(), isEmpty()
- entrySet(): return an iterable collection of the entries in M
- keySet(): return an iterable collection of the keys in M
- values(): return an iterator of the values in M

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Hash Tables

- Balanced BST (e.g., AVL trees): $O(\log N)$ for insertion, deletions and searches.
- Hashing is a technique used for performing insertions, deletions and searches in *constant average* time (i.e., $O(1)$ expected time)
- A hash table data structure consists of:
  - Hash function ***h***
  - Array of size ***N*** (bucket array)

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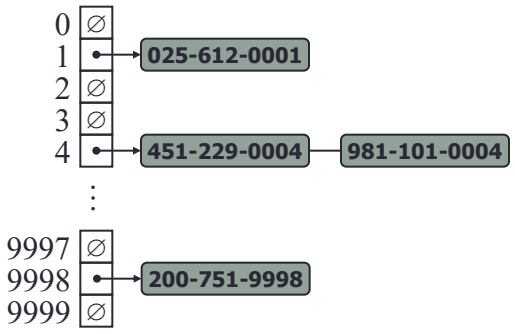
Example

- We design a hash table for a dictionary storing items (SIN, Name), where SIN (social insurance number) is a ten-digit positive integer
- Our hash table uses an array of size $N = 10{,}000$ and the hash function $h(x) = x \bmod N$
- We use chaining to handle collisions



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Hash Functions and Hash Tables

- A hash function ***h*** maps keys of a given type to integers in a fixed interval [0, ***N*** - 1]
- Example:
  h(***x***) = ***x*** mod ***N***
  is a hash function for integer keys
- The integer ***h***(***x***) is called the hash value of key ***x***
- The goal of a hash function is to uniformly disperse keys in the range [0, ***N*** - 1]
- A hash table for a given key type consists of
  - Hash function ***h***
  - Array of size ***N***
- A collision occurs when two keys in the dictionary have the same hash value.
- Collision handing schemes:
  - Chaining: colliding items are stored in a sequence
  - Open addressing: the colliding item is placed in a different cell of the table

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Design Issues

- Hash functions
  - Converting a key to an index in the hash table
- Collision handling
  - Separate chaining
  - Probing (open addressing)
    - Linear probing
    - Quadratic probing
    - Double hashing
- Table size (should be a prime number)

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Hash Functions

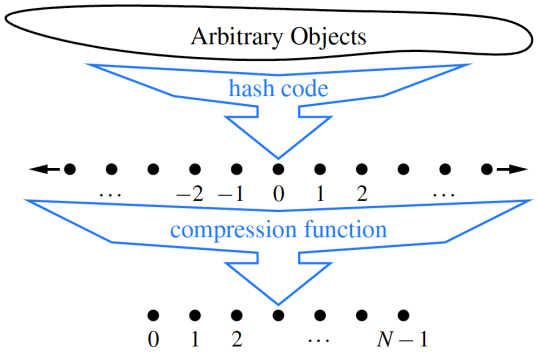

Figure 10.5: Two parts of a hash function: a hash code and a compression function.

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Hash Functions

- A hash function is usually specified as the composition of two functions:

  Hash code:

  h_1: keys $\rightarrow$ integers

  Compression function:

  h_2: integers $\rightarrow [0, N-1]$

- The hash code is applied first, and the compression function is applied next on the result, i.e.,

  $$h(x) = h_2(h_1(x))$$

- The goal of the hash function is to "disperse" the keys in an apparently random way.

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Hash Codes

- To "transform" an arbitrary key (e.g., words in an English dictionary) to an integer.

  h_1: keys $\rightarrow$ integers

- Should avoid collisions as much as possible

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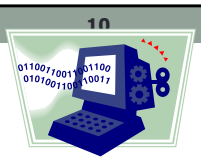
Hash Codes



- Treating the bit representation as an integer
  - We reinterpret the bits of the key as an integer
  - Suitable for keys of length less than or equal to the number of bits of the integer type (e.g., byte, short, int and float in Java)
    - Ex: Float.floatToIntBits(x)
- Component sum:
  - We partition the bits of the key into components of fixed length (e.g., 16 or 32 bits) and we sum the components (ignoring overflows), or XOR the components.
  - Suitable for numeric keys of fixed length greater than or equal to the number of bits of the integer type (e.g., long and double in Java)

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Hash Codes for Keys That Are Strings

- We need to convert a string to an integer before hashing.
- One option is to add up the ASCII values of the characters in the string.
  - Is this a good strategy?
- Polynomial accumulation:
  - We partition the bits of the key into a sequence of components of fixed length (e.g., 8, 16 or 32 bits).

    $x_0 x_1 \ldots x_{n-1}$
  - We evaluate the polynomial

    $p(z) = x_0 + x_1 z + x_2 z^2 + \ldots + x_{n-1} z^{n-1}$

    at a fixed value z, *ignoring overflows*.

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Polynomial Accumulation

- Polynomial $p(z)$ can be evaluated in $O(n)$ time using Horner's rule:
  - The following polynomials are successively computed, each from the previous one in $O(1)$ time

    $p_0(z) = x_{n-1}$

    $p_i(z) = x_{n-i-1} + z p_{i-1}(z)$

    $(i = 1, 2, \ldots, n-1)$
  - We have $p(z) = p_{n-1}(z)$
  - https://www.math10.com/en/algebra/horner.html
- Good z values: 33, 37, 39, 41.
  - Especially suitable for strings
    - $z = 33$ gives at most 6 collisions on a set of 50,000 English words.

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Hash Functions

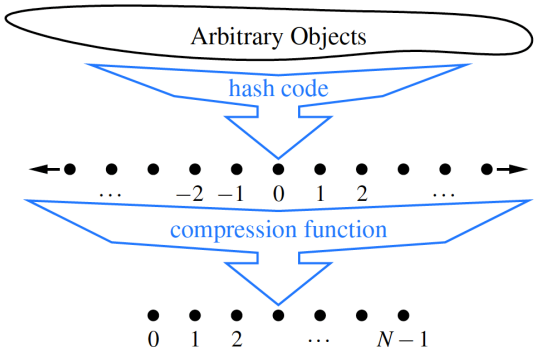

Figure 10.5: Two parts of a hash function: a hash code and a compression function.

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Compression Function: Division

- $h_2(y) = y \bmod N$
- The size N of the hash table is usually chosen to be a prime number to minimize the number of collisions.
- Example: inserting {200, 205, 210, 215, 220, . . . ,600} into a bucket array of size 100: each hash code collides with three others.
- Bucket array of size 101: no collision.
- Good compression function: probability of two different keys getting hashed to the same bucket is 1/N.

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Compression Function: MAD

- Multiply, Add and Divide (MAD):
  - $h_2(y) = [(ay + b) \bmod p] \bmod N$
  - N: size of the bucket array
  - p is a prime number larger than N
  - a and b: integers chosen at random interval $[0, p-1]$, with $a > 0$
- Helps eliminate repeated patterns in a set of integer keys

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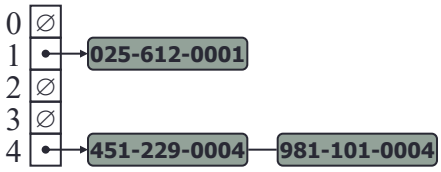
Collision Handling

- Collisions occur when different elements are mapped to the same cell.
- **Separate Chaining**: let each cell in the table point to a linked list of entries that map there



- Separate chaining is simple, but requires additional memory outside the table

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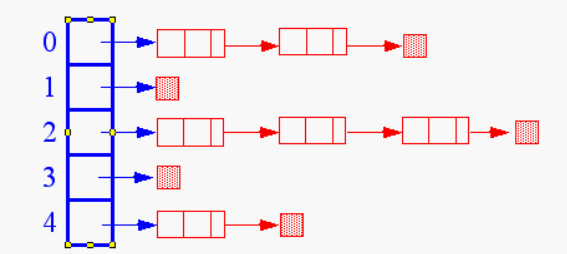
Separate Chaining



- Use ***chaining*** to set up ***lists*** of items with same index
- The ***expected*** search/insertion/removal time is O(n/N), provided that the indices are uniformly distributed
  - N = hash table size
  - n = number of elements in the table
- If n/N is *O*(1), the expected running time is *O*(1)

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Load Factor – Separate Chaining

- Define the load factor $\lambda = n/N$
  - n = number of elements in the hash table
  - N = hash table size (prime number)
- To obtain best performance with separate chaining, ensure $\lambda < 0.9$.
- As long as λ is *O*(1), insert, delete and search operations run in *O*(1) expected time.
- As we add more elements to the hash table, λ goes up $\Rightarrow$ rehashing (allocate a bigger table, define a new compression function, and put the elements into the new array).

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Next lecture: Collision Handling

- Separate chaining
- Probing (open addressing): the colliding item is placed in a different cell of the table
  - Linear probing
  - Quadratic probing
  - Double hashing