BINARY SEARCH TREES (11.1)

EECS 2011

Example Application

- Application: database of employee records
  - keys: social insurance numbers
  - add employee based on key
  - remove employee using key
  - search employee using key

Data Structure Choices

Operation	Doubly Linked List (unsorted)	Array (unsorted)
add	O( )	O( )
remove	O( )	O( )
search	O( )	O( )

Operation	Doubly Linked List (sorted)	Array (sorted)
add	O( )	O( )
remove	O( )	O( )
search	O( )	O( )

Map ADT (10.1.1)

- The Map ADT models a searchable collection of key-value items
- The main operations of a map are searching, inserting, and deleting items
- Keys must be unique.
- Applications:
  - credit card database
  - SIN database
  - student/employee database

We are interested in the following Map ADT methods:

- get(k): if the map has an item with key k, returns its value, else, returns NULL
- put(k, e): inserts item (k, e) into the map
- remove(k): if the map has an item with key k, removes it from the dictionary and returns its value, else returns NULL
- size(), isEmpty()

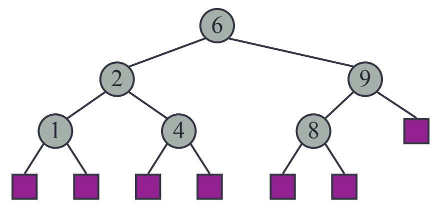
Binary Search Trees

- A binary search tree is a binary tree storing keys (or key-element pairs) at its internal nodes and satisfying the following property:

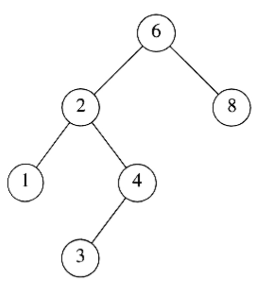
  Let u, v, and w be three nodes such that u is in the left subtree of v and w is in the right subtree of v. We have $key(u) \leq key(v) \leq key(w)$

- External nodes (dummies) do not store items (non-empty proper binary trees, for coding simplicity)

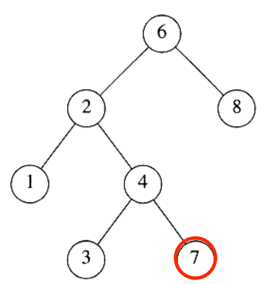
- An inorder traversal of a binary search trees visits the keys in increasing order
- The left-most child has the smallest key
- The right-most child has the largest key



Example of BST



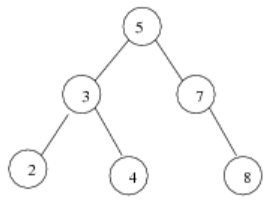
A binary search tree

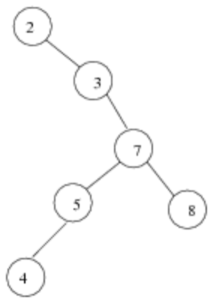


Not a binary search tree

More Examples of BST

The same set of keys may have different BSTs.

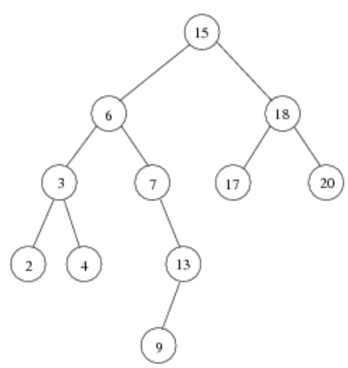




- Average depth of a node is O(logN).
- Maximum depth of a node is O(N).
- Where is the smallest key? largest key?

Inorder Traversal of BST

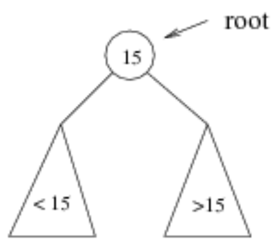
- Inorder traversal of BST prints out all the keys in sorted order.



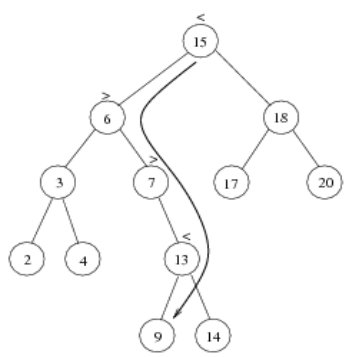
Inorder: 2, 3, 4, 6, 7, 9, 13, 15, 17, 18, 20

Searching BST

- If we are searching for 15, then we are done.
- If we are searching for a key < 15, then we should search in the left subtree.
- If we are searching for a key > 15, then we should search in the right subtree.



Example: Search for 9 ...



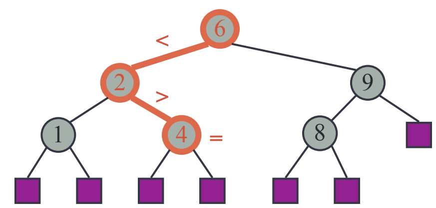
Search for 9:

1. compare 9:15(the root), go to left subtree;
2. compare 9:6, go to right subtree;
3. compare 9:7, go to right subtree;
4. compare 9:13, go to left subtree;
5. compare 9:9, found it!

Search Algorithm

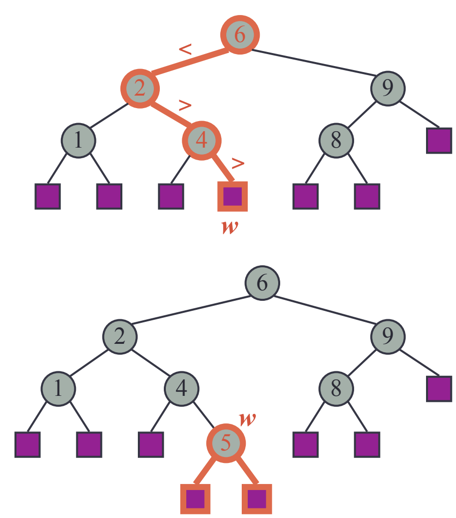
- To search for a key ***k***, we trace a downward path starting at the root
- The next node visited depends on the outcome of the comparison of ***k*** with the key of the current node
- If we reach a leaf, the key is not found and we return v (where the key should be if it will be inserted)
- Example:
  TreeSearch(4, *root*())
- Running time: ?

```
Algorithm TreeSearch( k, v )
   if isExternal (v)
      return (v);     // or return NO_SUCH_KEY
   if k < key(v)
      return TreeSearch( k, left(v) )
   else if k = key(v)
      return v
   else { k > key(v) }
      return TreeSearch( k, right(v) )
```



Insertion (distinct keys)

- To perform operation put(k, e), we first search for key k
- Assume k is not already in the tree, and let w be the leaf reached by the search
- We insert k at node w and expand w into an internal node using *expandExternal*(w, (k, e))
- Example:
  expandExternal(w, (5, e)) with e having key 5
- Running time: ?



Insertion Algorithm (distinct keys)

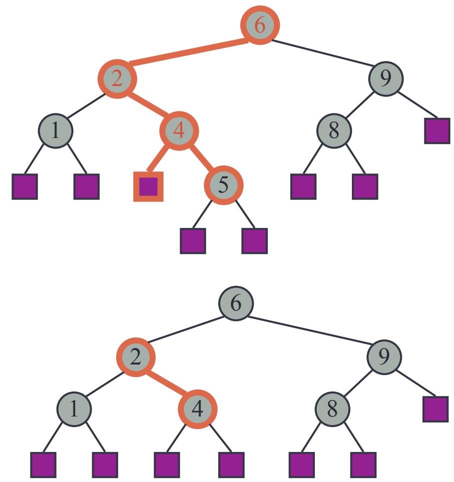
```
Algorithm TreeInsert( k, e ) {
   w = TreeSearch( k, root( ) );
   if ( k == key(w) )
      change w's value to e;
  else
      expandExternal( w, (k, e) );
}

Algorithm expandExternal( w, k, e ) {
  if ( isExternal( w ) {
     make w an internal node, store k and e into w;
     add two dummy nodes as w' s children;
  }  else  { error condition };
}
```

Deletion

- To perform operation *remove*(k), we first search for key k
- Assume key k is in the tree, and let v be the node storing k
- Three cases:
  - Case 1: v has no internal children
  - Case 2: v has exactly one internal child
  - Case 3: v has two internal children

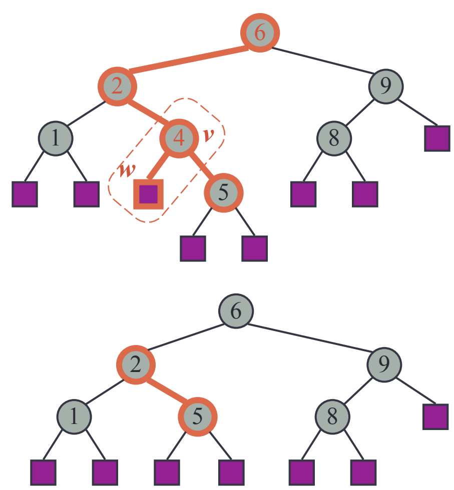
Deletion: Case 1

- Case 1: v has no children
- We simply remove v and its 2 dummy leaves.
- Replace v by a dummy node.
- Example: remove 5



Deletion: Case 2

- Case 1: v has exactly one child
- v's parent will "adopt" v's child.
- We connect v's parent to v's child, effectively removing v and the dummy node w from the tree.
- Example: remove 4

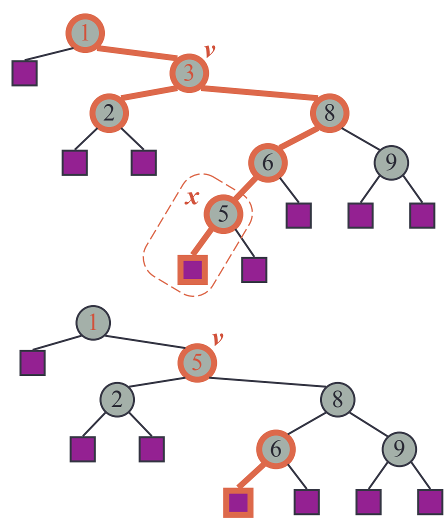


Deletion: Case 3

- Case 3: v has two children (and possibly grandchildren, great-grandchildren, etc.)
- Identify v's "heir": *either* one of the following two nodes:
  - the node x that immediately precedes v in an inorder traversal (right-most node in v's left subtree)
  - the node x that immediately follows v in an inorder traversal (left-most node in v's right subtree)
- Two steps:
  - copy content of x into node v (heir "inherits" node v);
  - remove x from the tree (use either case 1 or case 2 above).

Deletion: Case 3 Example

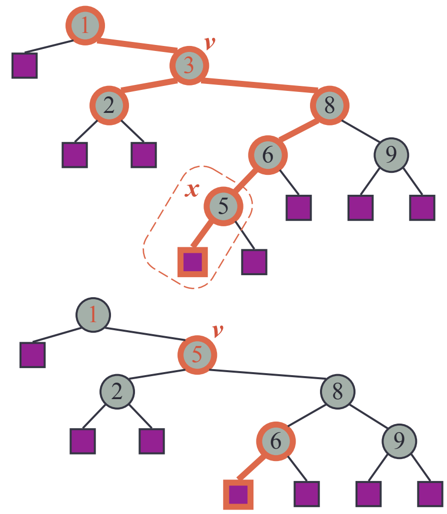
- Example: remove 3
- Heir = ?



- Running time of deletion algorithm: ?

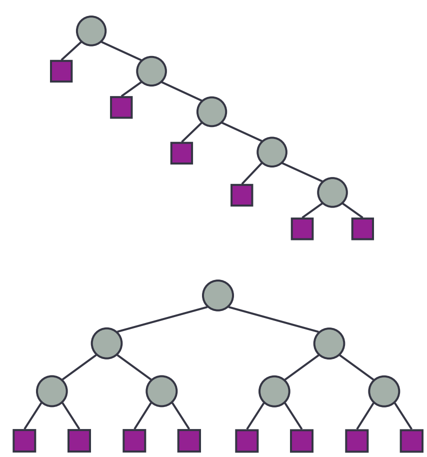
Deletion: Case 3 Steps

- Two steps of case 3:
  - copy content of x into node v (heir "inherits" node v);
  - remove x from the tree
    - if x has no child: call case 1
    - if x has one child: call case 2
    - x cannot have two children (why?)



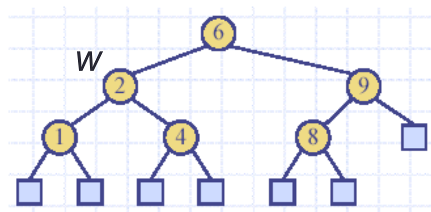
Performance

- Consider a map with n items implemented by means of a binary search tree of height h
  - the space used is $O(n)$
  - methods get(k) , put(k,e) and remove(k) take $O(h)$ time
- The height h is $O(n)$ in the worst case and $O(\log n)$ in the best case



Appendix: Insertion with Duplicate Keys

- To perform operation *put*(*k*, *e*), we first search for key *k*
- Assume *k* is already in the tree, for example, *k* = 2
- Let *w* be the node returned by *TreeSearch*(*k*, *root*( ))

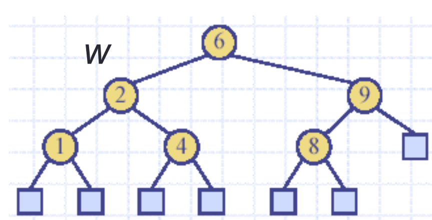


Insertion (duplicate keys)

- Call *TreeSearch*(*k*, *left*(*w*)) to find the leaf node for insertion
- Can insert to either the left subtree or the right subtree
- Call *TreeSearch*(*k*, *right*(*w*)) to insert to the right subtree
- If there are more duplicate keys in the subtree, call *TreeSearch* for each key found, until reaching a leaf node for insertion.

Running time: ?

Note: if inserting the duplicate key into the *left* subtree, keep searching the *left* subtree after a key has been found.



Summary

- Methods get(k) , put(k,e) and remove(k) take $O(h)$ time.
- The insertion order and removal order determine h.
- The height h is
  - $O(n)$ in the worst case
  - $O(\log n)$ in the best case
- Need self-balanced trees to achieve $O(\log n)$ time.

Next lecture …

- AVL trees (11.3)