EECS 2001N: Introduction to the Theory of Computation
Tutorial 2, Jan 17, 2:30 pm
Problems

1. Give a recursive definition of the set defined below:
$$S = \{a^n bc^n | n \in \mathbb{N} \cup \{0\}\}$$
2. Suppose a language L is defined recursively as:
$\epsilon \in L$,
for every x, y in L, $axby$ and $bxay$ are both in L,
nothing else is in L.

   Prove that L is precisely the set of strings in $\{a, b\}^*$ with equal numbers of a'a and b's.
3. Design a DFA for the language that contains only all binary strings of length 3.
4. Design a DFA for the language that contains only binary strings that end in 0110.
5. Design a DFA for the language that contains only binary strings of non-zero length whose bits sum to a multiple of 3.
6. Design a DFA for the language over $\Sigma = \{a, b\}$ that contains all words containing the string $abab$.
7. Design a DFA for the language over $\Sigma = \{a, b\}$ that contains all words **not** ending in aab.
8. Design a DFA for the language over $\Sigma = \{a, b\}$ that contains all words in which the third letter from the right is b.
9. Design a DFA for the language that contains only binary strings in which every odd position is a 1.
10. Design a DFA for the language over $\Sigma = \{a, b, c\}$ that contains all words in which there are an odd number of a's.
11. Design a DFA for the language that contains only binary strings in which the first and last symbols are different.
12. Consider the alphabet $\Sigma = \{a, b\}$. Design a DFA for the language
$L = \{w||w| > 0,$ and the difference in the number of a's and b's is even$\}$.
13. Consider the alphabet $\Sigma = \{a, b\}$. Design a DFA for the language
$L = \{w||w| > 0,$ and w has an even number of a's and an odd number of b's$\}$.

14. (*) Show that if L is a regular language, then so is $L' = \{w | w \in L \text{ and } w \in L^R\}$.

15. (*) Given a DFA, how can you determine if the language it accepts is finite or infinite?