EECS 2001N : Introduction to the Theory of Computation

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Course page: `http://www.eecs.yorku.ca/course/2001N`
Also on Moodle

Context Free Language Problems

- $A_{CFG} = \{\langle G, w\rangle | G \text{ is a CFG that generates } w\}$
- $E_{CFG} = \{\langle G\rangle | G \text{ is a CFG with } L(G) = \emptyset\}$
- $EQ_{CFG} = \{\langle G, H\rangle | G, H \text{ are CFGs with } L(G) = L(H)\}$

Acceptance of Context Free Languages

Recall: Chomsky Normal Form

- A CFG $G = (V, \Sigma, R, S)$ is in CNF if every rule is of the form
  $A \rightarrow BC$, or
  $A \rightarrow x$
  with variables $A \in V$ and $B, C \in V \backslash \{S\}$, and $x \in \Sigma$
  For the start variable S we also allow $S \rightarrow \epsilon$

- Chomsky NF grammars are easier to analyze

- The derivation $S \Rightarrow^* w$ requires $2|w|{-}1$ steps (apart from $S \rightarrow \epsilon$)

Acceptance of Context Free Languages - 2

The language

$$A_{CFG} = \{\langle G, w\rangle | G \text{ is a CFG that generates } w\}$$

is TM-decidable.
Proof: Perform the following algorithm:

- Check if G and w are proper, if not "reject"
- Rewrite G to G' in Chomsky normal form
- Take care of $w = \epsilon$ case via $S \to \epsilon$ check for G'
- List all G' derivations of length $2|w|-1$
- Check if w occurs in this list:
  if so "accept"; if not "reject"

Emptiness of Context Free Languages

The language

$$E_{CFG} = \{\langle G \rangle | G \text{ is a CFG with } L(G) = \emptyset\}$$

is TM-decidable.
Proof: Perform the following algorithm:

- Check if G is proper, if not "reject"
- Let $G = (V, \Sigma, R, S)$, define set $T = \Sigma$
- Repeat $|V|$ times:
  Check all rules $B \to X_1 \ldots X_k$ in R
  If $B \notin T$ and $X_1 \ldots X_k \in T^k$ then add B to T
- If $S \in T$ then "reject", otherwise "accept"

Equality of Context Free Languages

Is the language

$$EQ_{CFG} = \{\langle G, H\rangle | G, H \text{ are CFG's with } L(G) = L(H)\}$$

TM-decidable?

- For DFA's we could use the emptiness decision procedure to solve the equality problem
- For CFG's this is not possible because CFGs are not closed under complementation or intersection
- We suspect this problem is undecidable, but need machinery to **prove** this