Test 1

First Name: _____

Last Name: _____

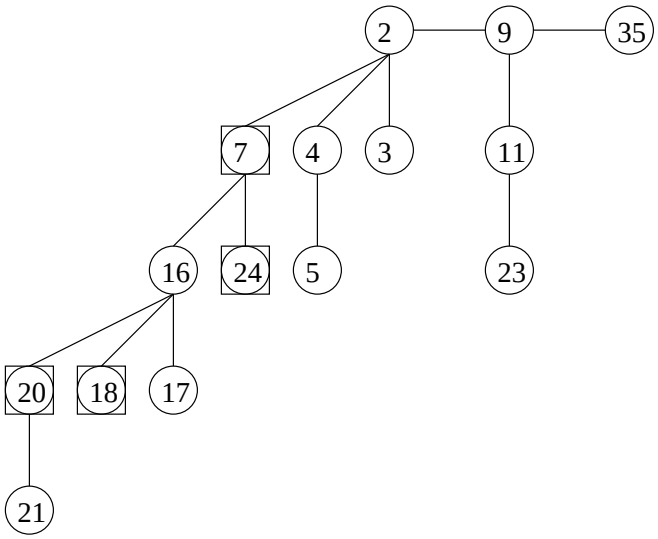
Student Number: _____

This test lasts 75 minutes.
Aids allowed: one 8.5×11 *inch piece of paper with handwritten notes.*
Make sure your test has 6 pages, including this cover page.
Answer in the space provided. (If you need more space, use page 6 and indicate clearly that your answer is continued there.)
Write legibly.

Question 1	/2
Question 2	/3
Question 3	/2
Question 4	/2
Question 5	/2
Question 6	/7
Question 7	/6
Total	/24

[2] **1.** What is the difference between an abstract data type and a data structure?

[3] **2.** The picture below shows a Fibonacci heap. Marked nodes are indicated by a square around the node. Show what the data structure would look like after the node with key 21 is deleted. Indicate which nodes are marked. Show your work.



[2] **3.** Binomial heaps and Fibonacci heaps are two data structures for priority queues. What are two important differences between them?

[2] **4.** Suppose a node of a Fibonacci heap has 52 children. What is the smallest possible number of grandchildren it can have? Briefly justify your answer.

[2] **5.** Willemena wants to implement a resizable array that allows INCREMENT-SIZE operations (but not DECREMENT-SIZE). Whenever the array becomes full, she creates a new larger array and copies all elements to it. Instead of doubling the size, she changes the size from n to $n + \lceil \sqrt{n} \rceil$. This way, the amount of wasted space is $O(\sqrt{n})$ instead of $O(n)$.

What is the total running time for a sequence of n INCREMENT-SIZES, starting with an array of size 1? Express your answer using Θ notation.

[7] **6.** Suppose you want to implement the union-find abstract data type using a collection of binomial heaps. Each binomial heap will contain all the elements of one of the disjoint sets.

(a) Briefly describe how you would implement the MAKE-SET, FIND-SET and UNION operations. State the worst-case running time of each operation in terms of n, the total number of elements in all the disjoint sets.

(b) Suppose you used your data structure in part (a) to implement Kruskal's algorithm for finding a minimum spanning tree of a graph. What would the running time be for an input graph with n nodes and m edges? State your answer using Θ notation. Briefly justify your answer.

[6] **7.** Consider a data structure that maintains a linked list of integers. It has two operations. APPEND(x) adds the integer x to the end of the list in $O(1)$ time. CHOP discards the oldest third of the list (i.e., if the list contains r elements, it discards the $\lceil r/3 \rceil$ elements closest to the front of the list) and prints the remaining elements in sorted order. (A CHOP does not change the order of the remaining elements in the linked list: it copies them to an array, sorts them in the array and then prints the array.) When the list contains r elements, a CHOP operation runs in $O(r \log r)$ time. Assume the linked list is initially empty.

(a) Consider a sequence of m operations that starts with n APPENDS followed by $m - n$ CHOPS. What is the total running time of this sequence? Express your answer in terms of m and n using Θ notation.

(b) Give a good upper bound on the total running time of any sequence of m operations that includes n APPENDS. Express your answer in terms of m and n using big-O notation. Prove your answer is correct.

This nearly blank page is just for additional workspace, if you need it.