Design Theory

1. Keys & FDs
2. The Normal Forms
3. Reasoning with FDs
4. Normalization

Table of Contents

Parke Godfrey

2016-10-05 initial [v1] 2016-10-17 [v2]

Acknowledgments

Thanks

- to *Jeffrey D. Ullman*
  for initial slidedeck
- to *Jarek Szlichta*
  for the slidedeck with significant refinements on which this is derived

1. Keys & FDs

Notation: *functionally determines*

Let $\mathcal{R}$ be the *set* of attr's of table $\mathbf{R}$.

If subset of attr's $\mathcal{K} \subseteq \mathcal{R}$ is “the” *key* of $\mathbf{R}$, then there is *at most* one tuple with given values for the attr's in $\mathcal{K}$ in (any instance of) table $\mathbf{R}$.

Another way to think of this is that, given values for $\mathcal{K}$, there is a distinct value for each attr. in $\mathcal{R} - \mathcal{K}$.

In this case, we say that $\mathcal{K}$ *functionally determines* $\mathcal{R}$. We denote this by

$$\mathcal{K} \mapsto \mathcal{R}$$

Notation: *keys & superkeys*

Given $\mathcal{R}$ — the set of all attr's of table $\mathbf{R}$ — we call *any* subset $\mathcal{S} \subseteq \mathcal{R}$ such that $\mathcal{S} \mapsto \mathcal{R}$ a *superkey* of table $\mathbf{R}$.

If no proper subset of a superkey $\mathcal{K}$ for $\mathbf{R}$ is also a superkey for $\mathbf{R}$ — that is, $\neg\exists\mathcal{J} \subset \mathcal{K}.\ \mathcal{J} \mapsto \mathcal{R}$ — then we call $\mathcal{K}$ a *key* of table $\mathbf{R}$.

Notation: *functional dependencies*

We might happen to know that, in some domain, $\mathcal{X} \mapsto \mathcal{Y}$ *holds*, where $\mathcal{X} \cup \mathcal{Y} \subset \mathcal{R}$, but $\mathcal{X} \not\mapsto \mathcal{R}$; that is, $\exists \mathrm{A} \in \mathcal{R} - (\mathcal{X} \cup \mathcal{Y}).\ \mathcal{X} \not\mapsto \{\mathrm{A}\}$.

Thus, $\mathcal{X}$ is *not* a superkey of $\mathbf{R}$! But such things as $\mathcal{X} \mapsto \mathcal{Y}$ will be important.

We will call $\mathcal{X} \mapsto \mathcal{Y}$ a *functional dependency* (“FD”).

Note. Call an FD a *superkey FD if* its left-hand side is a superkey; call it a *key FD if* its LHS is a key.

Not all FDs are superkey FDs!

Canonical form

splitting right-hand sides

Consider $\mathcal{X} \mapsto \mathcal{Y}$ where $\mathcal{Y} = \{\mathrm{Y}_0, \ldots, \mathrm{Y}_{\mathrm{k}-1}\}$. Then that FD is equivalent to the set of FDs

$$\forall i \in \{0, \ldots, k-1\}.\ \mathcal{X} \mapsto \{\mathrm{Y}_\mathrm{i}\}$$

We generally express FD's with singleton right-hand sides.

> There is no *splitting rule* for left-hand sides! That would be incorrect.

Shorthand for FDs

As shorthand, instead of using “{”'s and “}”'s everywhere, when we are using single letters for attr's, we just munge them together.

E.g., we write $\{A, B, C\} \mapsto \{D, E\}$ as $ABC \mapsto DE$.

Example: Drinker & FDs

$$\textbf{Drinker}(\text{name}, \text{address}, \text{beer}, \text{manf}, \text{favBeer})$$

Here, we mean the drinker (name) likes that beer, and it is manufactured by manf.

Reasonable FDs to assert:

- $\text{name} \mapsto \text{addr}, \text{favBeer}$

  Equivalent to

  - $\text{name} \mapsto \text{addr}$
  - $\text{name} \mapsto \text{favBeer}$

- $\text{beer} \mapsto \text{manf}$

Example: key of Drinker

$\{\text{name}, \text{beer}\}$ is a *key* of **Drinker** because neither $\{\text{name}\}$ nor $\{\text{beer}\}$ is a superkey.

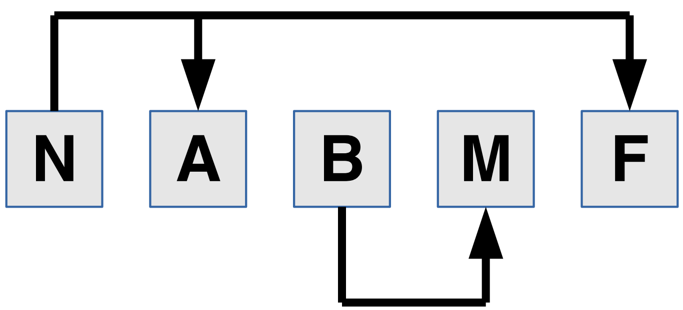
- $\text{name} \not\mapsto \text{manf}$
- $\text{beer} \not\mapsto \text{addr}$

In this case, there are no other keys.

But there are lots of superkeys! Namely, any superset of $\{\text{name}, \text{beer}\}$.

Visualizing

We sometimes draw out FDs to see what is going on.



Where do FDs come from?

- The designer *presribes* them by naming keys.
- From real-world “constraints” that the designer knows and *prescribes*.

  E.g., “no two classes can meet in the same room at the same time”

$$\text{hour}, \text{room} \mapsto \text{class}$$

Because we may have FDs *in addition to* the prescribed key FDs, additional key FDs might exist.
We may have to *assert* our FDs, then *deduce* the keys by systematic exploration!

Problem: non-key FDs

Non-key FDs are problematic, because they allow for the potential of *anomalies*.

---

our game: Design to have only superkey FDs.

Anomalies / redundancies

- **update anomaly**: one occurrence of a fact is changed, but not all occurrences are.
- **deletion anomaly**: a valid fact is lost when a tuple is deleted.

The *goal* of relational schema design is to avoid such anomalies and redundancy.

Non-key FDs cause these problems because these violate our “single source of truth” mandate.

Example: anomalies

- **deletion anomaly**.
  - No drinker likes the beer *Bud*.
  - Thus, no tuple appears in $\mathbf{Drinker}$ with $\text{beer} = \text{'Bud'}$.
  - Then we do not have the information that $\text{beer} = \text{'Bud'}$ and $\text{manf} = \text{'Anheuser Busch'}$.
- **insertion anomaly**.
  - Say there is a tuple in $\mathbf{Drinker}$ with $\text{beer} = \text{'Bud'}$ and $\text{manf} = \text{'Anheuser Busch'}$ (with, say, $\text{name} = \text{'Jeff'}$). *correct*
  - Someone accidentally adds another tuple later with $\text{beer} = \text{'Bud'}$ and $\text{manf} = \text{'Labatt'}$ (with, say, $\text{name} = \text{'Franck'}$). *incorrect*
  - **Note**. Our key of $\{\text{name}, \text{beer}\}$ has *not* been violated.
  - Who manufactures *Bud*?

2. The Normal Forms

First pass

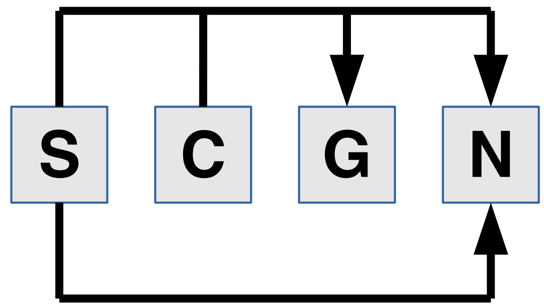
What the normal forms do

1. **1NF**: Every table (relation) has a key *prescribed*. (*Trivial* to achieve. Why?)
2. **2NF**: Every table is in 1NF *and* no table has a *partial-key dependency*.
3. **3NF**: Every table is in 2NF *and* no table has a *transitive dependency*.
4. **BCNF**: Every table is in 3NF *and* no table has a *back dependency*.

If we can achieve BCNF, we should be good to go!

Partial key dependencies (¬2NF)

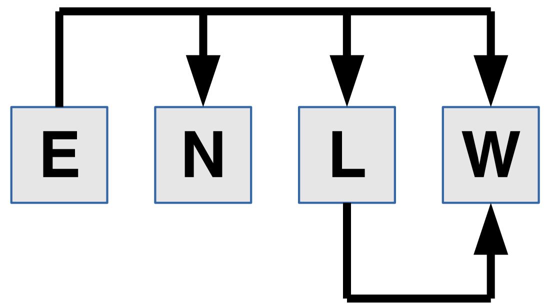
Consider **Enrol**(s#, c#, grade, student_name) (SCGN).



- “The” key FD is clearly $\mathrm{SC} \mapsto \mathrm{GN}$.
- We also have the non-key FD $\mathrm{S} \mapsto \mathrm{N}$.
  - We call this a *partial-key* dependency because its LHS is a proper subset of *a* key.
    E.g., $\{\mathrm{S}\} \subset \{\mathrm{S}, \mathrm{C}\}$.

Transitive dependencies (¬3NF)

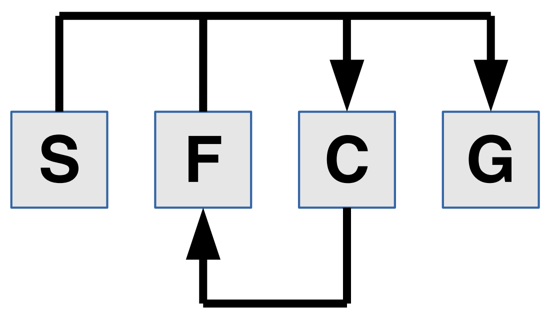
Consider **Emp**(emp#, name, level, wage) (ENLW).



- "The" key FD is $\mathrm{E} \mapsto \mathrm{NLW}$.
- We also have the non-key FD $\mathrm{L} \mapsto \mathrm{W}$.
  - We call this a *transitive* dependency because its LHS and RHS are not subsets of *any* keys. E.g., $\{\mathrm{L}\} \nsubseteq \{\mathrm{E}\}$ and $\{\mathrm{W}\} \nsubseteq \{\mathrm{E}\}$.

Back dependencies (¬BCNF)

Consider **Enrol$_2$**(st#, fac#, class#, grade) (SFCG).



- "The" key FD is $SF \mapsto CG$.
- We also have the non-key FD $C \mapsto F$.
  - We call this a *back* dependency because its LHS *seems* to not be a subset of *a* key but its RHS is part of a key.
    E.g., $\{C\} \subseteq \{SF\}$.

Second pass

Formally defining things

But your “definitions” above seem fuzzy!
“*Seems* to not be a subset of *a* key?!”

The examples above give you a *feel* for what is going on.

Why are they not *formal* definitions?

We did not say what the situation is when

- the LHS or RHS of a non-key FD overlaps with a key, *or*
- there is *more* than one key.

What were the keys in $\mathbf{Enrol_2}$?

Well, $\{\mathrm{S}, \mathrm{F}\}$. That was stated!

But also $\{\mathrm{S}, \mathrm{C}\}$!

But then... wouldn't $\mathrm{C} \mapsto \mathrm{F}$ be a partial-key dependency?!

Formal definitions

- **1NF**: Each attr. is an elementary type. A key is defined for each relation.
- **2NF**: Whenever $\mathcal{X} \mapsto \mathrm{A}$ holds in $\mathbf{R}$ and $\mathrm{A} \notin \mathcal{X}$,
  - A is *prime*, or
  - $\mathcal{X}$ is not a proper subset of *any* key of $\mathbf{R}$.
- **3NF**: Whenever $\mathcal{X} \mapsto \mathrm{A}$ holds in $\mathbf{R}$ and $\mathrm{A} \notin \mathcal{X}$,
  - A is *prime*, or
  - $\mathcal{X}$ is a superkey of $\mathbf{R}$.
- **BCNF**: Whenever $\mathcal{X} \mapsto \mathrm{A}$ holds in $\mathbf{R}$ and $\mathrm{A} \notin \mathcal{X}$,
  - $\mathcal{X}$ is a superkey of $\mathbf{R}$.

An attr. is *prime* if it is part of *any* key of $\mathbf{R}$. E.g., $\mathrm{name} \in \{\mathrm{name}, \mathrm{beer}\}$ in $\mathbf{Drinker}$.

3. Reasoning with FDs

Some FDs that hold for a relation may be *implicit*.

That is, an implicit FD may logically follow from the set of known FDs.

We will need to *infer* the implicit ones. For example, we need to know a relation's keys to test whether it is in BCNF.

Is this *hard*? Well…*yes* and *no*.

Why might it be hard?

How many different keys can a single-key rel'n have?

Exponential!

Consider a rel'n with n attr's. Then, there are 2^n possible different keys.

How many possible keys of length k?

$$\binom{n}{k}$$

Why might it be hard?

How many keys might a rel'n have simultaneously?

There can be lots of FDs! And lots of key FDs too.

Consider I know that $A_1 \mapsto A_2$, $A_2 \mapsto A_1$, …, $Z_1 \mapsto Z_2$, $Z_2 \mapsto Z_1$.

There are 2^{26} keys!

There can be an *exponential* number of key FDs.

What is easy?

From a set of attr's $\mathcal{X}$, I can find the *closure*, $\mathcal{X}^+$, of that set of attr's easily.

(In fact, there is a *linear* runtime algorithm known for this!)

FD axiomatization

A *sound* and *complete* set of axioms for FDs is as follows.

1. *Reflexivity*. $\mathcal{X} \mapsto \mathcal{Y}$ if $\mathcal{Y} \subseteq \mathcal{X}$.
    - These are called *trivial* FDs.
2. *Augmentation*. If $\mathcal{X} \mapsto \mathcal{Y}$ then $\mathcal{X} \cup \mathcal{Z} \mapsto \mathcal{Y} \cup \mathcal{Z}$ for any $\mathcal{Z}$.
3. *Transitivity*. If $\mathcal{X} \mapsto \mathcal{Y}$ and $\mathcal{Y} \mapsto \mathcal{Z}$ then $\mathcal{X} \mapsto \mathcal{Z}$.

Computing $\square^+$

Computing the *closure* of a set of attr's $\mathcal{X}$ is just the *fixpoint* with respect to *transitivity* then.

Let $\mathcal{X}_0 = \mathcal{X}$.

Define $\mathcal{X}_{i+1} = \mathcal{X}_i \cup \bigcup_{\mathcal{F}_i} \mathcal{Z}$.

where $\mathcal{F}_i = \{\mathcal{Y} \mapsto \mathcal{Z} \mid \mathcal{Y} \subseteq \mathcal{X}_i\}$.

Then $\mathcal{X}^+ = \mathcal{X}_i$ for which $\mathcal{F}_i = \{\}$.

Ever need to find *all* keys of a rel'n?

Unfortunately, we might have to.

For instance, to check a *schema* for meeting normal form.

See Exercise #9 from *Exercises for Study* (and *with answers*).

4. Normalization

Goal: A “correct” schema in BCNF.
Or in 3NF, if that is not possible.

So…how to achieve BCNF (or 3NF)?

Two approaches

decomposition: a *top-down* approach

- Start with our schema and *refine* it.

synthesis: a *bottom-up* approach

- Start with the prescribed FDs and *create* a schema from them.

Decomposition

Method Sketch

1. Find a *problematic* FD; i.e., one that breaks BCNF.
2. Make the problematic FD “go away”. How?
   - Let the FD be key of its own rel'n.
   - That is, split the non-BCNF rel'n into *two* rel'ns in a *lossless* way.
3. Repeat until no more problematic FDs!

See Exercise #16 from *Exercises for Study* (and *with answers*).

Two goals

1. ***lossless* decomposition**
    - We must ensure that the *final* schema can “reproduce” our *original* schema. That nothing is *lost*.
2. **dependency preservation**
    - *All* of our FDs are ensured by the *final* schema (by rel'n's keys).

Lossless join decomposition

Say we have a rel'n schema $\mathbf{R}$ of ABCDE and an FD $\mathrm{B} \mapsto \mathrm{E}$ that violates BCNF for $\mathbf{R}$.

We can break $\mathbf{R}$ into *two* rel'ns:

1. BE
   - This is just the “FD” itself!
   - Its key can be B, the LHS of the FD.
2. ABCD
   - And the rest of what is left over from $\mathbf{R}$
   - …repeating the LHS of the FD!
     This will server as a foreign key from 2) to 1).
   - The key can be whatever was key for $\mathbf{R}$.

Lossless join decomposition (general)

Given $\mathbf{R}$ and an FD $\mathcal{X} \mapsto \mathcal{Y}$ violating $\mathbf{R}$ — assume that $\mathcal{X} \mapsto \mathcal{Y}$ is non-trivial, that $\mathcal{X} \cap \mathcal{Y} = \{\}$ — replace $\mathbf{R}$ by two rel'ns:

1. $\mathcal{X} \cup \mathcal{Y}$
2. $\mathcal{R} - \mathcal{Y}$

What goes wrong if not *lossless*?

We cannot reproduce the “database” with respect to the original *schema* from our *decomposed* schema.

That is, if we “joined” our two decomposed rel'ns, we might not recover the original rel'n.

Example of a bad decomposition

Consider ABC with no FDS and I break it into AB and BC . Let our table be

A	B	C
1	2	3
4	2	5

Example of a bad decomposition (2)

This decomposes into

A	B
1	2
4	2

and

B	C
2	3
2	5

Example of a bad decomposition (3)

But “joining” these back together ($\mathrm{AB} \bowtie \mathrm{BC}$) does *not* give us the same thing that was in $\mathbf{R}$!

A	B	C
1	2	3
1	2	5
4	2	3
4	2	5

The extra resulting tuples here from the “lossy” join are called *spurious*.

But what about *dependency preservation*?

good news

- Lossless join decomposition steps can always get us eventually to BCNF!

bad news

- the *resulting* schema may not be dependency preserving.

See Exercise #17 from *Exercises for Study* (and *with answers*).

Decomposition

Revised Method

1. Find a *problematic* FD for a rel'n, decompose the rel'n losslessly with respect to the FD.
2. Repeat until no more problematic FDs!
3. Add back any FDs that are not covered in the *resulting* schema as rel'ns.

Can we always have BCNF?

The method above often works, but does not always.

What can go wrong? Some of the non-covered FDs that we add back in as rel'ns may not be in BCNF!

- Well, we could decompose an added, non-BCNF rel'n…
- But then the corresponding FD is not covered *again*!
- *Stuck*.

The result *does* arrive always to a 3NF, dependency preserving schema!

Is there just *one* lossless decomposition?

Of course not! The decomposition depends on the *order* of the decomposition steps we apply.

There may be an *exponential* number of lossless-join decompositions.

One of them might be BCNF and dependency preserving (after we add back in the non-covered)!

- But finding this one might be *hard*.
- And it might not even exist!

Synthesis Method

Find a *minimal basis* of the set of declared (explicit) FDs.

That's it!

Is polynomial!

It is *polynomial* to find *a* minimal basis!

The resulting schema *is* in 3NF and it *is* dependency preserving.

Note. There is no notion of “lossless” here; there is no “original” schema to compare against.

Finding *all* minimal bases, though, is *exponential*.

- One of them might be in BCNF too!
- But this is *NP-complete* to find.

Minimal basis

1. Throw away any FD that can be *derived* from the others (the remaining ones)

   Repeat until no such FD remains.

2. Throw away any attr. on the LHS of an FD *if* the resulting FD plus the others can *derive* the original FD.

   Repeat until no such FD remains.

See Exercise #18 from *Exercises for Study* (and *with answers*).