EECS 3101 M: Design and Analysis of Algorithms

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Course page: `http://www.eecs.yorku.ca/course/3101M`
Also on Moodle

Definitions - 1

- $G = (V, E)$, V = set of nodes/vertices, E = set of edges
- Edges incident on a vertex
- Adjacent vertices
- degree of a node
- neighborhood of a node
- Self-loop

Definitions - 2

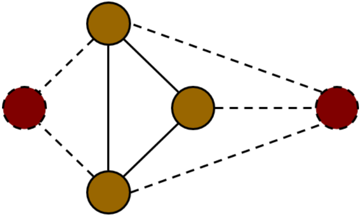
- Edge Types:
  - Directed edge: ordered pair of vertices (u, v)
    - u : origin, v : destination
  - Undirected edge: unordered pair of vertices (u, v)
- Graph Types:
  - Directed graph: all the edges are directed
  - Undirected graph: all the edges are undirected
- Paths:
  - Simple Paths
  - Cycles
  - Simple cycles: no vertex repeated

Elementary Properties

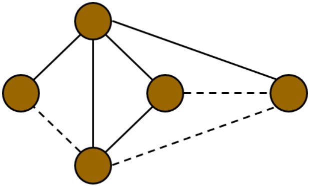
- The sum of degrees is even (equals twice the number of edges in an undirected graph)
- The sum of indegrees equals sum of outdegrees in a directed graph
- In an undirected graph $m \leq \frac{n(n-1)}{2}$
  What is the bound for directed graphs?

Subgraphs

- A subgraph S of a graph G is a graph such that
  - The vertices of S are a subset of the vertices of G
  - The edges of S are a subset of the edges of G
- A spanning subgraph of G is a subgraph that contains all the vertices of G



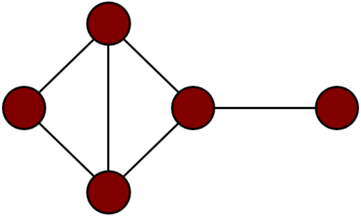
Subgraph



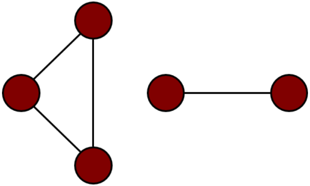
Spanning subgraph

Connected graphs

- A graph is connected if there is a path between every pair of vertices
- A connected component of a graph G is a maximal connected subgraph of G



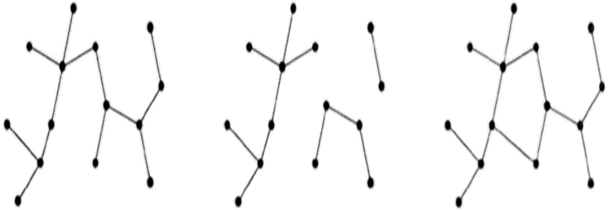
Connected graph



Disconnected graph with two connected components

Trees

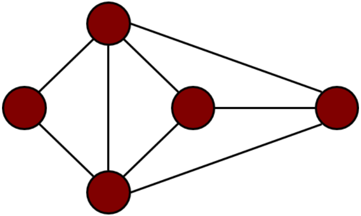
- A tree is a connected, acyclic, undirected graph
- A forest is a set of trees (not necessarily connected)



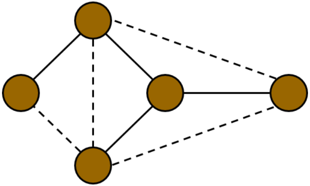
Tree, forest, a cyclic graph

Spanning Trees

- A spanning tree of a connected graph is a spanning subgraph that is a tree
- A spanning tree is not unique unless the graph is a tree
- Spanning trees have applications to the design of communication networks
- A spanning forest of a graph is a spanning subgraph that is a forest



graph



Spanning tree

Graph Problems

- Connectivity: Are all vertices reachable from each other?
- Reachability: Is a node v reachable from a node u?
- Shortest Paths
- (Sub)graph Isomorphism
- Graph Coloring
- And many others

Coloring graphs

Basic idea:

- Assign colors to nodes
- Each edge should connect nodes of **different** colors
- Want to minimize the number of colors used
- The minimum number of colors is the property of a graph, called **chromatic number**

Bipartite graphs

- The set of vertices V can be partitioned into disjoint sets V_1, V_2 such that all edges go between V_1, V_2
- A graph is bipartite **if and only if** it is 2-colorable
- How do we know if a graph is 2-colorable?

Greedy Bipartite Graph Coloring - idea

Assumes a connected undirected graph

- start at any node and color it red; label it "finished"
- color its neighbours blue and label the nodes "started"
- consider any node labeled "started".
- if it has a neighbour with the same color, exit with the message "not bipartite"
- else color its uncolored neighbours with the opposite color and label them "started"; label the current node "finished"

Greedy Bipartite Graph Coloring - Correctness

Part 1: If the algorithm fails the graph is not 2-colorable

- if the graph contains an odd cycle, it cannot be 2-colorable

- if the algorithm fails, the graph contains an odd cycle Why did the algorithm fail to 2-color? 2 nodes joined by the edge had the same color. So the distances from the least common ancestor of the 2 nodes to the nodes are both even or both odd. Adding the edge between them creates an odd cycle

Part 2: If the algorithm succeeds the graph is 2-colorable

More on Graph Coloring

- Determining if a graph has chromatic number of 1 or 2 is easy
- Determining if a graph has chromatic number 3 is NP-complete (believed to be intractable)
- For special classes of graphs, the chromatic number is known.
- For planar graphs the chromatic number is 4