EECS 3101 Winter '19: Design and Analysis of Algorithms

Assignment 3

Weight: 6%, Due: April 3, 11:59 pm

Notes:

1. The assignment MUST be typed, and submitted as a .pdf file.
2. Use the moodle link to submit your assignments. No late submissions will be accepted. Please do not send files by email.
3. You must do this assignment individually.
4. Submit this assignment only if you have read and understood the policy on academic honesty on the course web page. If you have questions or concerns, please contact the instructor.
5. If you want an extension, talk to the instructor.

Problem 1

[5 points] Suppose you are given an undirected graph G and a start node s. Your task is to design an algorithm that returns FALSE if G is not a tree and returns TRUE and labels each vertex v with the number of nodes in the subtree rooted at v if G is a tree. Note that the orientation of edges is implicit given the start node.

Hint: Modify DFS to solve the problem.

Problem 2

[4 points] Consider the MST of a connected undirected graph. Prove or provide a counterexample for the claim that the path between any 2 nodes in the MST is the shortest path between those nodes in the original graph. Does your answer change if the MST is unique?

Problem 3

[5+1 points] A city has k hospitals and n houses. Your job is to label each house with the distance to the nearest hospital. You are given a graph as input. The graph has houses and hospitals as nodes (hospitals are marked with a special label to distinguish them from houses) and edges are put between nodes when there is a road connecting two nodes.

Assume that each edge has cost 1. Design a $O(|E| + |V|)$ algorithm for solving the problem.

Does your algorithm work correctly if the edges have weights that are arbitrary positive integers? Justify your answer.