EECS 1028 M: Discrete Mathematics for Engineers

Suprakash Datta
Office: LAS 3043

Course page: http://www.eecs.yorku.ca/course/1028
Also on Moodle
Check your understanding

Let \( A = \{0, 1\} \).

1. What is \( \emptyset - A \)?

2. What is \( A \times \emptyset \)?

3. What is \( \emptyset \times \emptyset \)?

4. What is \( \mathcal{P}(\emptyset) \)?

Answer key:
1. \( \emptyset \)
2. \( \emptyset \)
3. \( \emptyset \)
4. \( \{\emptyset\} \)
Claim: $0.999999999999999 \ldots = 1$.

Proof: Let $x = 0.999999999999999 \ldots$. Therefore,

$$10x = 9.999999999999999 \ldots$$

$$10x - x = 9$$

$$x = 1$$

Note: You may recognize that $x = \frac{9}{10} + \frac{9}{100} + \ldots$. This is a geometric series and we computed the sum above.
Another Example

Claim: Let $n \in \mathbb{N}$. If $n^2$ is even, then $n$ is even.

Proof: Suppose this is false. Then $n$ is odd. But $n^2 = n \times n$ must be odd because the product of 2 odd numbers is odd. That is a contradiction!

Why? We assumed that $n^2$ is even but now it turns out that $n^2$ is odd.

Something went wrong! Our algebra was correct, so our original assumption (that $n$ is odd) is incorrect.

Therefore $n$ cannot be odd, or it must be even.

Note: There are several other ways to prove this.
Proof by contradiction:
Let’s suppose the statement is false; i.e., $\sqrt{2}$ is a rational number.
Then
$\sqrt{2} = a/b$ where $a, b$ are integers, $b \neq 0$.
We ALSO assume that $a/b$ is simplified to lowest terms, i.e., $gcd(a, b) = 1$. So,

$$\sqrt{2} = a/b \text{ squaring}$$
$$2 = a^2/b^2, \text{ or}$$
$$a^2 = 2b^2$$

So $a^2$ is even implying that $a$ is also even (from the last slide)
Since $a$ is even, then $a = 2k$ for some integer $k$. 

Proof that $\sqrt{2}$ is not rational
Substituting $a = 2k$ we get:

\[
\begin{align*}
a^2 &= 2b^2 \\
4k^2 &= 2b^2 \\
b^2 &= 2k^2
\end{align*}
\]

So $b^2$ is even, implying $b$ is even (again from the last claim we proved).

We assumed that $\gcd(a, b) = 1$ but now it turns out that $a, b$ are both even, so $\gcd(a, b) \geq 2$. That is a contradiction.

So our original assumption (that $\sqrt{2}$ is rational) is incorrect.

\[
\text{Therefore } \sqrt{2} \text{ cannot be rational.}
\]