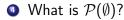
EECS 1028 M: Discrete Mathematics for Engineers

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Course page: http://www.eecs.yorku.ca/course/1028 Also on Moodle

Check your understanding

- Let $A = \{0, 1\}$.
 - What is $\emptyset A$?
 - **2** What is $A \times \emptyset$?
 - O What is Ø × Ø?



Answer key:

- 1. Ø
- 2. Ø
- 3. Ø
- 4. {Ø}

A Simple Proof

$$10x = 9.99999999999999...$$

$$10x - x = 9$$

$$x = 1$$

Note: You may recognize that $x = \frac{9}{10} + \frac{9}{100} + \dots$ This is a geometric series and we computed the sum above.

Another Example

<u>Claim</u>: Let $n \in \mathbb{N}$. If n^2 is even, then n is even.

<u>Proof:</u> Suppose this is false. Then *n* is odd. But $n^2 = n * n$ must be odd because the product of 2 odd numbers is odd. That is a contradiction!

Why? We assumed that n^2 is even but now it turns out that n^2 is odd.

Something went wrong! Our algebra was correct, so our original assumption (that n is odd) is incorrect.

Therefore n cannot be odd, or it must be even.

Note: There are several other ways to prove this.

Proof that $\sqrt{2}$ is not rational

Proof by contradiction:

Let's suppose the statement is false; i.e., $\sqrt{2}$ is a rational number. Then

 $\sqrt{2} = a/b$ where a, b are integers, $b \neq 0$. We ALSO assume that a/b is simplified to lowest terms, i.e., gcd(a, b) = 1. So,

$$\sqrt{2} = a/b$$
 squaring
 $2 = a^2/b^2$, or
 $a^2 = 2b^2$

So a^2 is even implying that a is also even (from the last slide) Since a is even, then a = 2k for some integer k.

Proof that $\sqrt{2}$ is not rational - Continued

Substituting a = 2k we get:

$$a^2 = 2b^2$$

$$4k^2 = 2b^2$$

$$b^2 = 2k^2$$

So b^2 is even, implying b is even (again from the last claim we proved).

We assumed that gcd(a, b) = 1 but now it turns out that a, b are both even, so $gcd(a, b) \ge 2$. That is a contradiction.

So our original assumption (that $\sqrt{2}$ is rational) is incorrect.

Therefore $\sqrt{2}$ cannot be rational.