EECS 1028 M: Discrete Mathematics for Engineers

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Course page: `http://www.eecs.yorku.ca/course/1028`
Also on Moodle

Check your understanding

Let $A = \{0, 1\}$.

1. What is $\emptyset - A$?

2. What is $A \times \emptyset$?

3. What is $\emptyset \times \emptyset$?

4. What is $\mathcal{P}(\emptyset)$?

Answer key:
1. $\emptyset$
2. $\emptyset$
3. $\emptyset$
4. $\{\emptyset\}$

A Simple Proof

Claim: $0.9999999999999999\ldots = 1$.

Proof: Let $x = 0.9999999999999999\ldots$. Therefore,

$$\begin{aligned} 10x &= 9.9999999999999999\ldots \\ 10x - x &= 9 \\ x &= 1 \end{aligned}$$

Note: You may recognize that $x = \frac{9}{10} + \frac{9}{100} + \ldots$. This is a geometric series and we computed the sum above.

Another Example

Claim: Let $n \in \mathbb{N}$. If n^2 is even, then n is even.
Proof: Suppose this is false. Then n is odd. But $n^2 = n * n$ must be odd because the product of 2 odd numbers is odd. That is a contradiction!
Why? We assumed that n^2 is even but now it turns out that n^2 is odd.

Something went wrong! Our algebra was correct, so our original assumption (that n is odd) is incorrect.

Therefore n cannot be odd, or it must be even.

Note: There are several other ways to prove this.

Proof that $\sqrt{2}$ is not rational

Proof by contradiction:
Let's suppose the statement is false; i.e., $\sqrt{2}$ is a rational number. Then
$\sqrt{2} = a/b$ where a, b are integers, $b \neq 0$.
We ALSO assume that a/b is simplified to lowest terms, i.e., $gcd(a, b) = 1$. So,

$$\begin{aligned} \sqrt{2} &= a/b \text{ squaring} \\ 2 &= a^2/b^2, \text{ or} \\ a^2 &= 2b^2 \end{aligned}$$

So a^2 is even implying that a is also even (from the last slide)
Since a is even, then $a = 2k$ for some integer k.

Proof that $\sqrt{2}$ is not rational - Continued

Substituting $a = 2k$ we get:

$$\begin{aligned} a^2 &= 2b^2 \\ 4k^2 &= 2b^2 \\ b^2 &= 2k^2 \end{aligned}$$

So b^2 is even, implying b is even (again from the last claim we proved).
We assumed that $gcd(a, b) = 1$ but now it turns out that a, b are both even, so $gcd(a, b) \geq 2$. That is a contradiction.

So our original assumption (that $\sqrt{2}$ is rational) is incorrect.

Therefore $\sqrt{2}$ cannot be rational.