EECS 1028 M: Discrete Mathematics for Engineers

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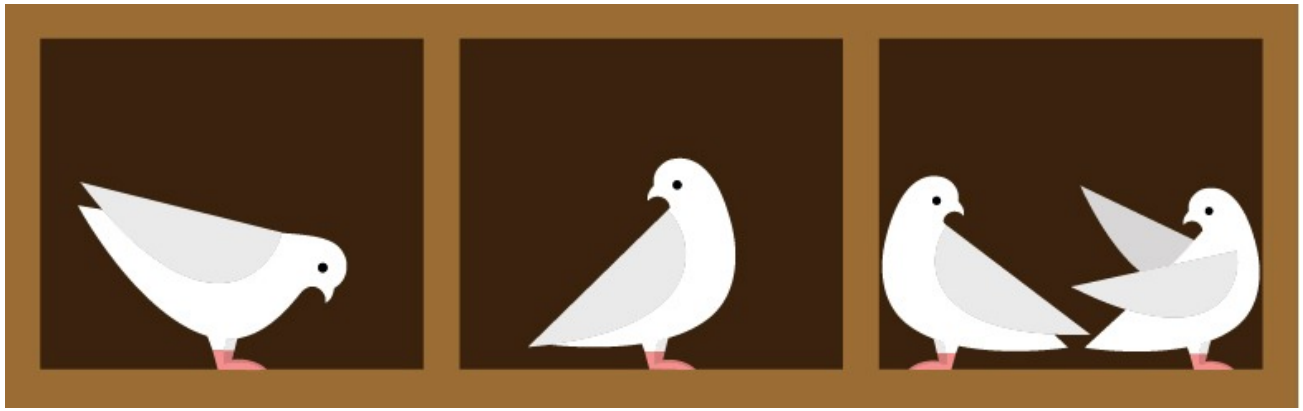
Course page: `http://www.eecs.yorku.ca/course/1028`
Also on Moodle

Pigeonhole Principle (Sec 6.2, excluding 6.2.3)



https://www.ethz.ch/en/news-and-events/eth-news/news/2016/05/creative-proofs-with-pigeons-and-boxes.html

Two statements:

- Pigeonhole Principle: If $n+1$ balls are distributed among n bins then at least one bin has more than 1 ball
- Generalized Pigeonhole Principle: If n balls are distributed among k bins then at least one bin has at least $\lceil n/k \rceil$ balls

Lots of interesting (and difficult) problems!

Pigeonhole Principle: Examples

- In any group of 367 people, at least 2 people must share a birthday
- In any group of 27 English words, at least 2 must start with the same letter
- In a class of 22 people, at least 2 must get the same score on a test out of 20, assuming all scores are integers

Generalized Pigeonhole Principle: Examples

- If there are 16 people and 5 possible grades, 4 people must have the same grade.

- There are 50 baskets of apples. Each basket contains no more than 24 apples. So there are at least 3 baskets containing the same number of apples.

Pigeonhole Principle: Difficulty

- May not be clear how to apply the principle
- Sometimes we need to cleverly design the pigeons and the holes
- If we do this correctly, the proof appears elegant; Otherwise, the problem seems impossibly hard

Pigeonhole Principle: More Difficult Problems

1. Prove that no straight line can be drawn such that it does not go through any of the vertices of a triangle but intersects all three sides of the triangle

- Fact: a straight line divides the plane into two half-planes
- By the Pigeonhole Principle, one half-plane must have at least 2 of the 3 vertices of the triangle.
- So the line does not cut the side joining these 2 vertices

Pigeonhole Principle: More Difficult Problems - 2

2. Show that among any 4 numbers one can find 2 numbers so that their difference is divisible by 3.

- Fact: any integer leaves a remainder of 0,1 or 2 when divided by 3
- By the Pigeonhole Principle, at least 2 of the 4 numbers must leave the same remainder
- The difference of these 2 numbers is divisible by 3

Generalization: Show that among any $n+1$ integers one can find 2 numbers so that their difference is divisible by n

Pigeonhole Principle: Inverted Problem

- If a Martian has an infinite number of red, blue, yellow, and black socks in a drawer, how many socks must the Martian pull out of the drawer to guarantee he has a pair?

- What is the minimum number of students, each of whom comes from one of the ten provinces, who must be enrolled at an university to guarantee that there are at least 100 who come from the same province?

Pigeonhole Principle: Exercises

- Pg 426-7, Q 4, 6, 17

- Very Challenging: Given 101 integers from $1, 2, \ldots, 200$, there are at least two integers such that one of them is divisible by the other