EECS 1028 M: Discrete Mathematics for Engineers

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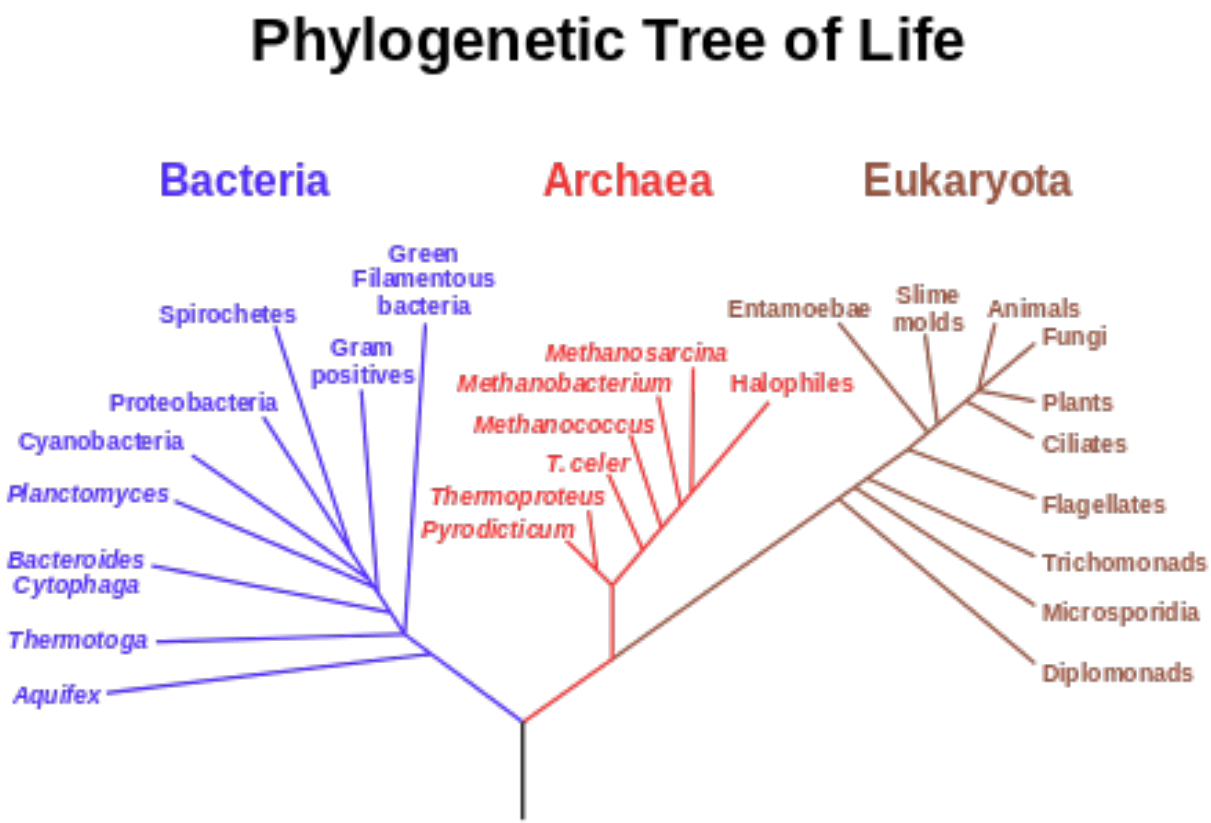
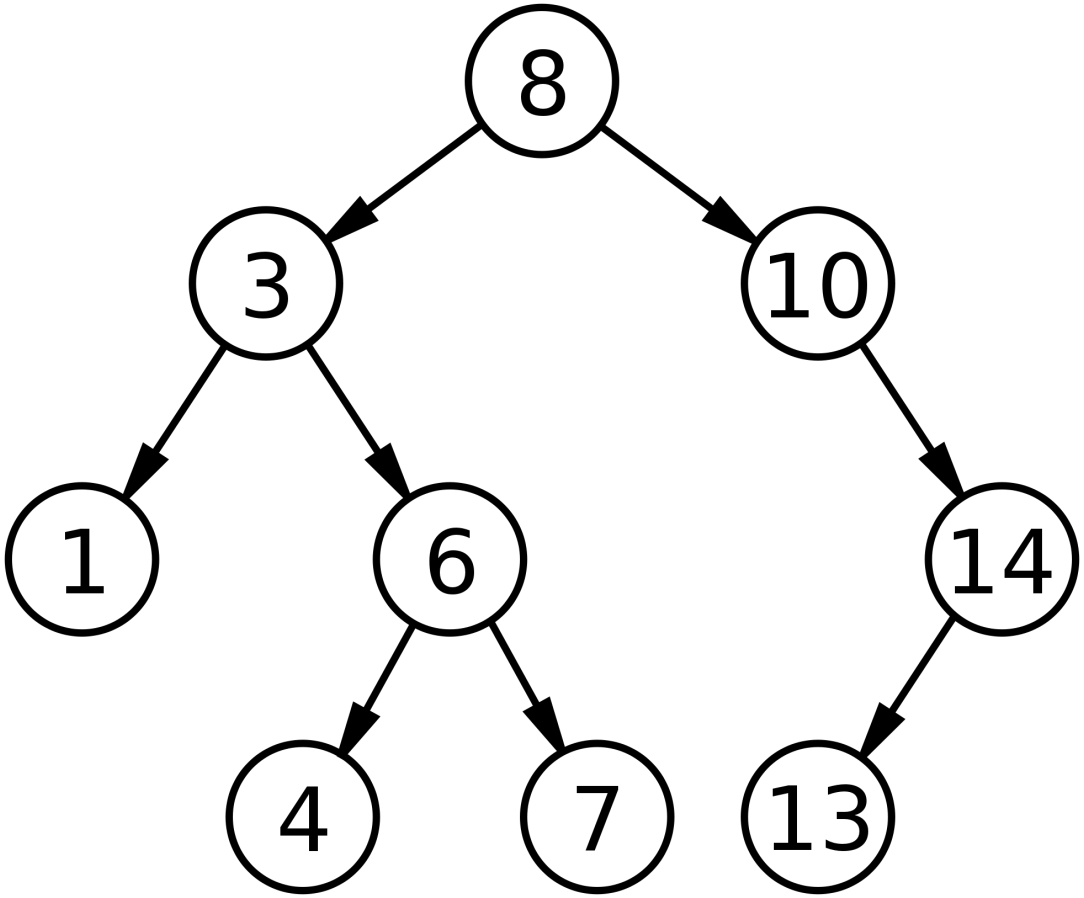
Course page: `http://www.eecs.yorku.ca/course/1028`
Also on Moodle

More on Trees

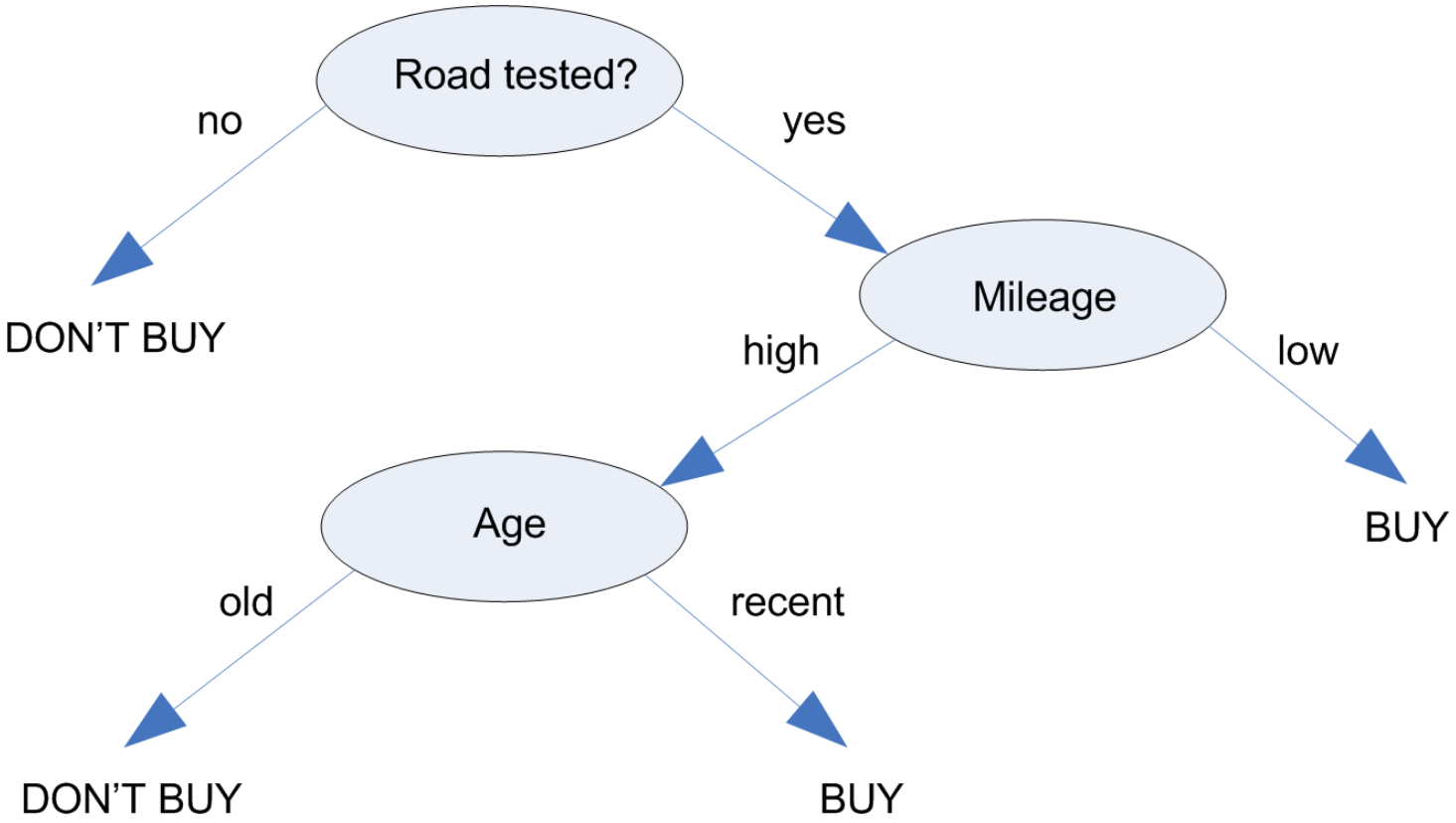
Sec 11.1



More on Trees: Binary Search Trees



More on Trees: Decision Trees



From `https:`

`//www.ibm.com/support/knowledgecenter/en/SS3RA7_15.0.0/com.ibm.spss.modeler.help/nodes_treebuilding.htm`

Other Examples of Trees

- Org charts
- Directory structures
- Organic molecules

Properties of Trees

- **Theorem** (page 746): An undirected graph is a tree iff there is a unique simple path between any two vertices
- **Theorem** (page 788): A tree with n nodes has $n - 1$ edges
- **Theorem** (page 789): A full m-ary tree with i internal vertices has $mi + 1$ vertices
- **Theorem** (page 790): There are at most m^h leaves in a m-ary tree of height h

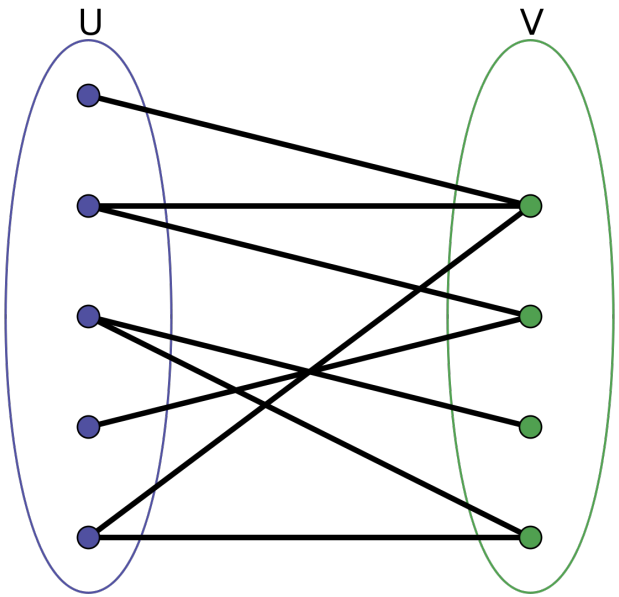
More on Graphs

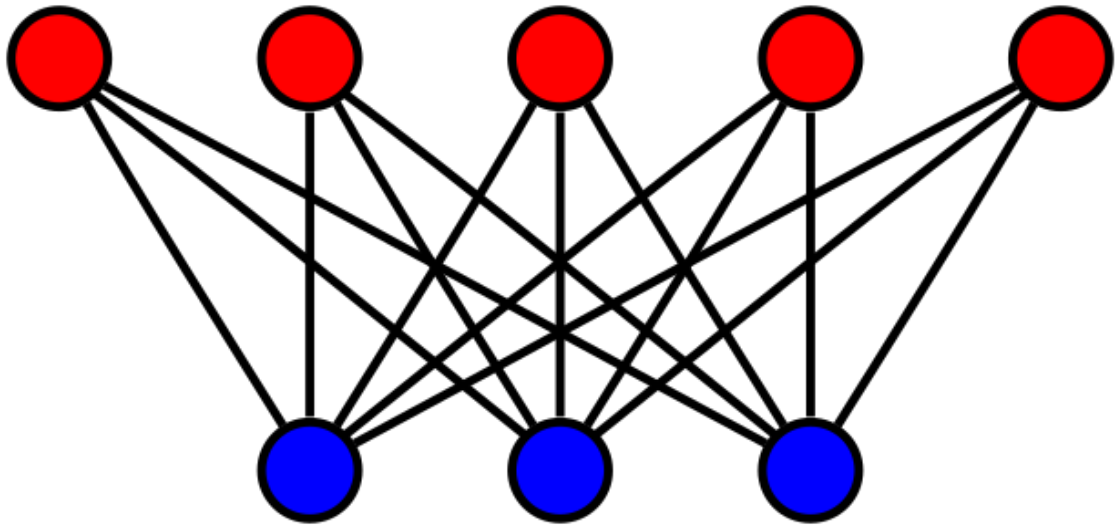
Sec 10.1, 10.2

- Some special families

- Computational problems on graphs

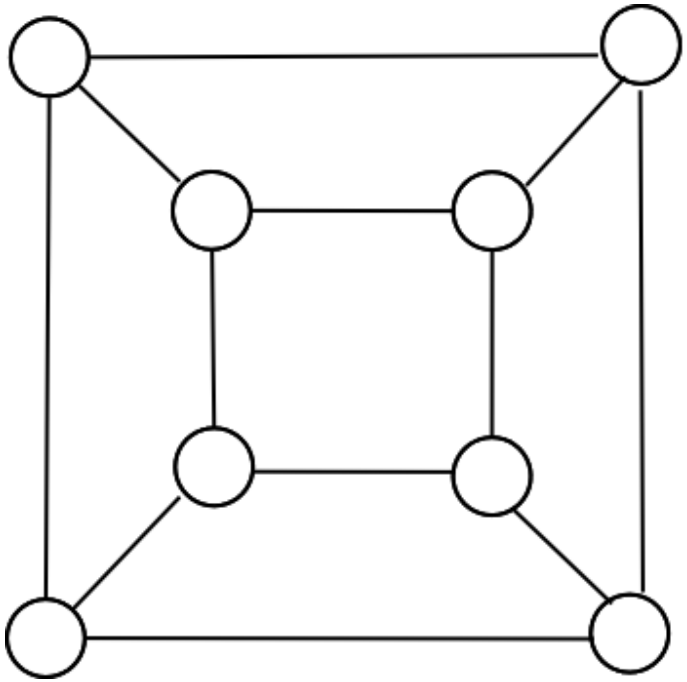
Bipartite Graphs



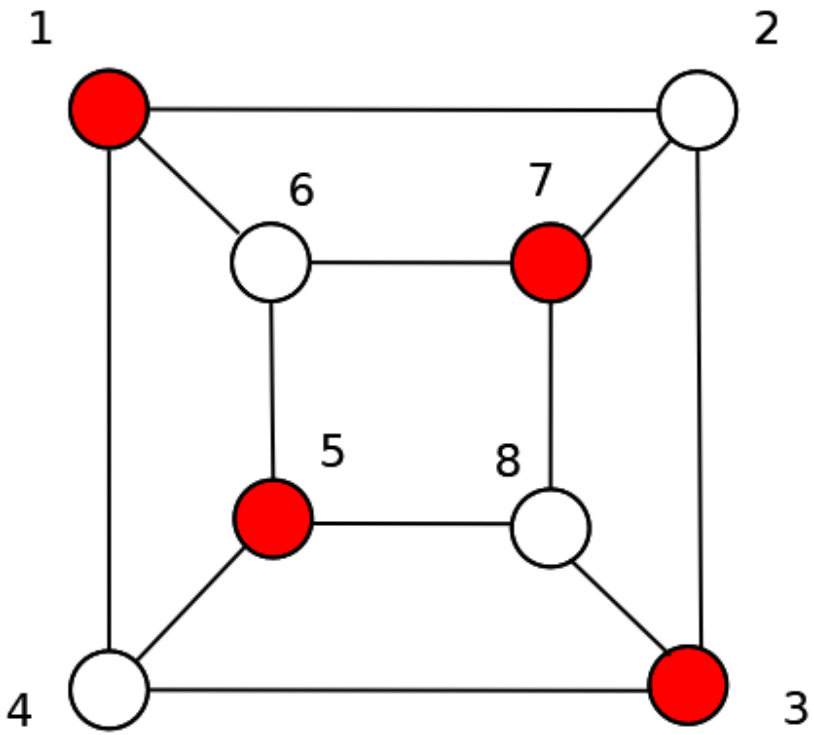


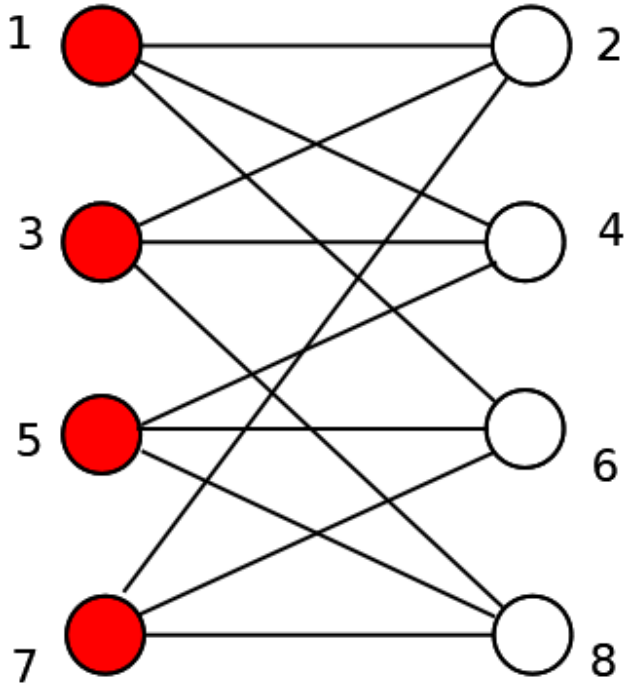
Bipartite Graphs - 2

- Vertex set V partitioned into V_1, V_2, all edges are between V_1, V_2
- How can you recognize a bipartite graph?



Bipartite Graphs - 3

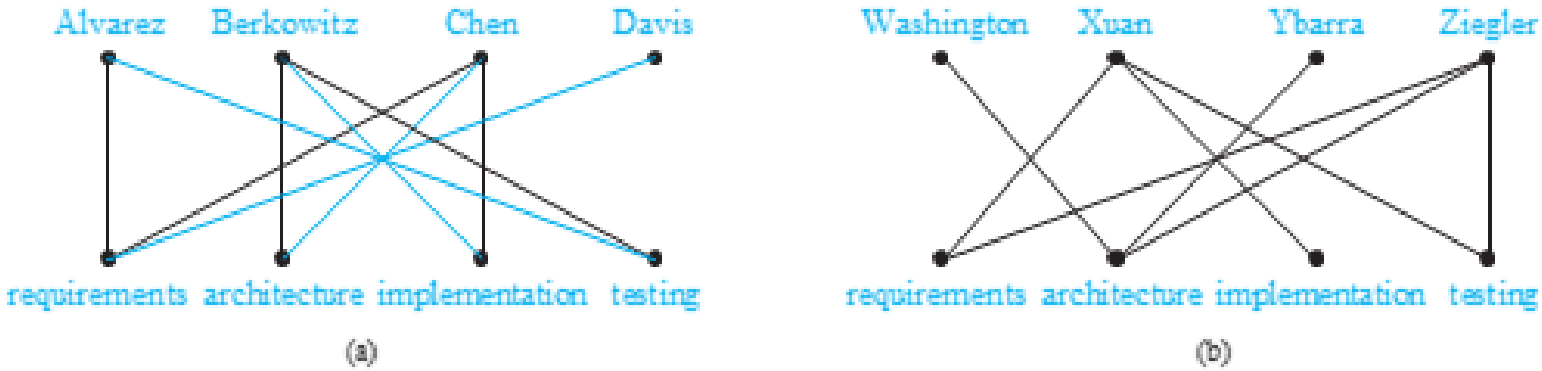




Bipartite Graphs - Coloring

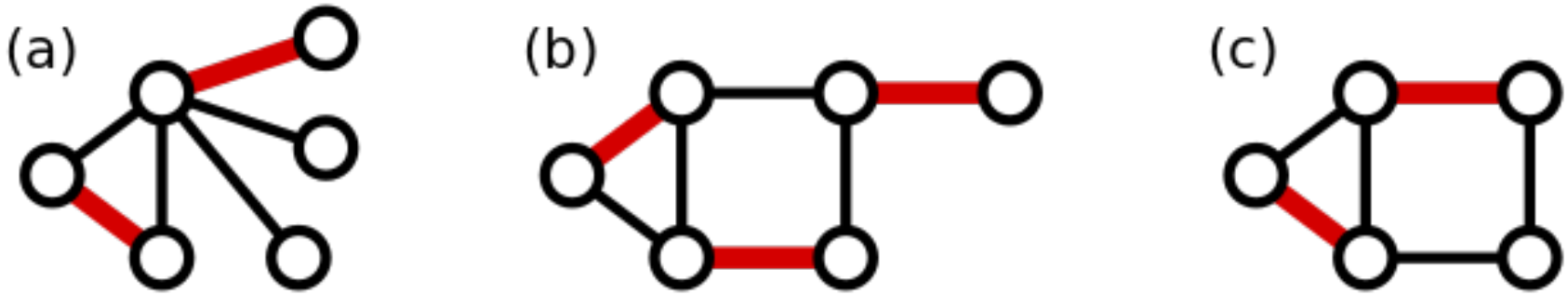
- **Theorem**(page 657): A graph is bipartite iff it is 2 colorable
- **Proof:** A given 2-coloring implies it is bipartite; it is easy to get a 2-coloring if it is known to be bipartite
- How to get a valid 2-coloring?
  Greedy algorithm: color a node white, its neighbours red and so on
- Proof of correctness: later courses
- Any graph containing an odd length cycle is **not** bipartite

Bipartite Graphs - Matching



- A matching is a subset of edges such that no 2 edges are incident on the same vertex
- A maximum matching is a matching with the maximum number of edges

General Graphs - Matching



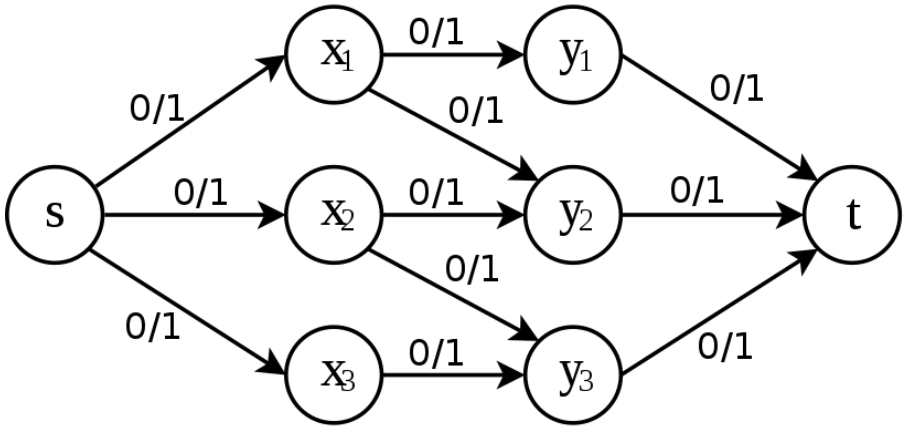
- Same definitions hold
- Are these graphs bipartite?

Bipartite Graphs - Maximum Matching

- When does a maximum matching possible?
- Let A be a set of vertices; $N(A)$ is the set of its neighbour vertices
- Hall's Theorem (page 659): A bipartite graph has a complete matching iff $|N(A)| \geq |A|$ for all subsets $A \subseteq V_1$

Bipartite Graphs - Finding a Maximum Matching

- Hall's Theorem (page 659) only guarantees the existence of a matching
- It does not yield an efficient algorithm to find a maximum matching
- Most commonly used algorithm uses Network Flow to find a maximal matching



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Other Graph Problems

- Connectivity
- Graph Isomorphism
- Graph Coloring