

CSE 3401: Intro to AI & LP Informed Search

- Required Readings: Chapter 3, Sections 5 and 6, and Chapter 4, Section 1.

Heuristic Search.

- In uninformed search, we don't try to evaluate which of the nodes on the frontier are most promising. We never "look-ahead" to the goal.
 - E.g., in uniform cost search we always expand the cheapest path. We don't consider the cost of getting to the goal.
- Often we have some other knowledge about the merit of nodes, e.g., going the wrong direction in Romania.

Heuristic Search.

- Merit of a frontier node: different notions of merit.
 - If we are concerned about the **cost of the solution**, we might want a notion of merit of how costly it is to get to the goal from that search node.
 - If we are concerned about **minimizing computation** in search we might want a notion of ease in finding the goal from that search node.
 - We will focus on the "**cost of solution**" notion of merit.

Heuristic Search.

- The idea is to develop a domain specific heuristic function $h(n)$.
- $h(n)$ **guesses** the cost of getting to the goal from node n .
- There are different ways of guessing this cost in different domains. I.e., heuristics are **domain specific**.

Heuristic Search.

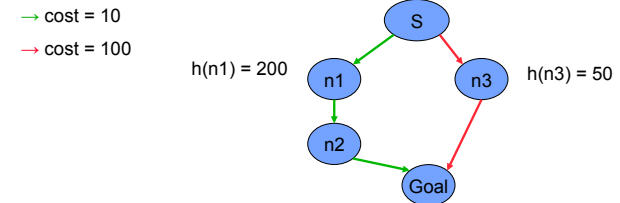
- Convention: If $h(n_1) < h(n_2)$ this means that we guess that it is cheaper to get to the goal from n_1 than from n_2 .
- We require that
 - $h(n) = 0$ for every node n that satisfies the goal.
 - Zero cost of getting to a goal node from a goal node.

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Using only $h(n)$ Greedy best-first search.

- We use $h(n)$ to rank the nodes on open.
 - Always expand node with lowest h -value.
- We are greedily trying to achieve a low cost solution.
- However, this method ignores the cost of getting to n , so it can be lead astray exploring nodes that cost a lot to get to but seem to be close to the goal:



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A* search

- Take into account the cost of getting to the node as well as our estimate of the cost of getting to the goal from n .
- Define
 - $f(n) = g(n) + h(n)$
 - $g(n)$ is the cost of the path to node n
 - $h(n)$ is the heuristic estimate of the cost of getting to a goal node from n .
- Now we **always expand the node with lowest f -value on the frontier.**
- The f -value is an estimate of the cost of getting to the goal via this node (path).

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Conditions on $h(n)$

- We want to analyze the behavior of the resultant search.
- Completeness, time and space, optimality?
- To obtain such results we must put some further conditions on the heuristic function $h(n)$ and the search space.

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Conditions on $h(n)$: Admissible

- $c(n1 \rightarrow n2) \geq \epsilon > 0$. The cost of any transition is greater than zero and can't be arbitrarily small.
- Let $h^*(n)$ be the cost of an optimal path from n to a goal node (∞ if there is no path). Then an **admissible** heuristic satisfies the condition
 - $h(n) \leq h^*(n)$
 - i.e. h always underestimates of the true cost.
- Hence
 - $h(g) = 0$
 - For any goal node "g"

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Consistency/monotonicity.

- Is a stronger condition than $h(n) \leq h^*(n)$.
- A **monotone/consistent** heuristic satisfies the triangle inequality (for all nodes $n1, n2$):

$$h(n1) \leq c(n1 \rightarrow n2) + h(n2)$$
- Note that there might be more than one transition (action) between $n1$ and $n2$, the inequality must hold for all of them.
- As we will see, monotonicity implies admissibility.

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Intuition behind admissibility

- $h(n) \leq h^*(n)$ means that the search won't miss any promising paths.
 - If it really is cheap to get to a goal via n (i.e., both $g(n)$ and $h^*(n)$ are low), then $f(n) = g(n) + h(n)$ will also be low, and the search won't ignore n in favor of more expensive options.
 - This can be formalized to show that admissibility implies optimality.

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Intuition behind monotonicity

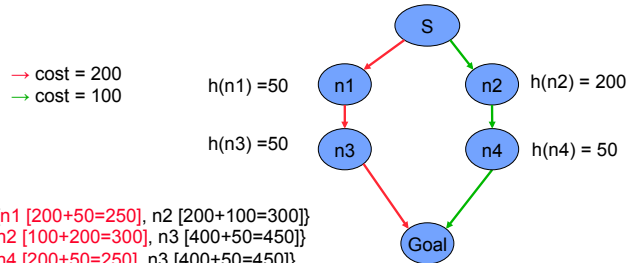
- $h(n1) \leq c(n1 \rightarrow n2) + h(n2)$
 - This says something similar, but in addition one won't be "locally" misled. See next example.

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Example: admissible but nonmonotonic

- The following h is not consistent since $h(n2) > c(n2 \rightarrow n4) + h(n4)$. But it is admissible.



{S} → {n1 [200+50=250], n2 [200+100=300]}
 → {n2 [100+200=300], n3 [400+50=450]}
 → {n4 [200+50=250], n3 [400+50=450]}
 → {goal [300+0=300], n3 [400+50=450]}

We **do find** the optimal path as the heuristic is still admissible. **But** we are misled into ignoring n2 until after we expand n1.

Monotonicity implies admissibility

Proof: by induction on number of steps to a goal node M.

- Base case: If n is a goal node, then $h(n) = 0 = h^*(n)$, so $h(n) \leq h^*(n)$.
- Induction step: Assume that $h(n_k) \leq h^*(n_k)$ if number of steps to goal at n_k is at most K . Show that the proposition must hold for nodes n_{k+1} where number of steps to goal is $K+1$.
 - Let n_k be the next node along a shortest path from n_{k+1} to goal
 - $h(n_{k+1}) \leq c(n_k \rightarrow n_{k+1}) + h(n_k)$, since h is monotone
 - $h(n_k) \leq h^*(n_k)$, by induction hypothesis
 - So $h(n_{k+1}) \leq c(n_k \rightarrow n_{k+1}) + h^*(n_k)$
 - Thus $h(n_{k+1}) \leq h^*(n_{k+1})$
- If goal is unreachable from a node n , then $h^*(n) = \infty$ and result trivially holds.

Consequences of monotonicity

- The f -values of nodes along a path must be non-decreasing.

- Let $\langle \text{Start} \rightarrow n_1 \rightarrow n_2 \dots \rightarrow n_k \rangle$ be a path. We claim that

$$f(n_i) \leq f(n_{i+1})$$

- Proof:**

$$\begin{aligned} f(n_i) &= c(\text{Start} \rightarrow \dots \rightarrow n_i) + h(n_i) \\ &\leq c(\text{Start} \rightarrow \dots \rightarrow n_i) + c(n_i \rightarrow n_{i+1}) + h(n_{i+1}) \\ &= c(\text{Start} \rightarrow \dots \rightarrow n_i \rightarrow n_{i+1}) + h(n_{i+1}) \\ &= g(n_{i+1}) + h(n_{i+1}) \\ &= f(n_{i+1}). \end{aligned}$$

Consequences of monotonicity

- If n_2 is expanded after n_1 , then $f(n_1) \leq f(n_2)$

Proof:

- If n_2 was on the frontier when n_1 was expanded,
 - $f(n_1) \leq f(n_2)$
 - otherwise we would have expanded n_2 .
- If n_2 was added to the frontier after n_1 's expansion, then let n be an ancestor of n_2 that was present when n_1 was being expanded (this could be n_1 itself). We have $f(n_1) \leq f(n)$ since A^* chose n_1 while n was present in the frontier. Also, since n is along the path to n_2 , by property (1) we have $f(n) \leq f(n_2)$. So, we have
 - $f(n_1) \leq f(n_2)$.

Consequences of monotonicity

3. When n is expanded every path with lower f -value has already been expanded.
 - Assume by contradiction that there exists a path $\langle \text{Start}, n_0, n_1, n_{i-1}, n_i, n_{i+1}, \dots, n_k \rangle$ with $f(n_k) < f(n)$ and n_i is its last expanded node.
 - Then n_{i+1} must be on the frontier while n is expanded:
 - a) by (1) $f(n_{i+1}) \leq f(n_k)$ since they lie along the same path.
 - b) since $f(n_k) < f(n)$ so we have $f(n_{i+1}) < f(n)$
 - c) by (2) $f(n) \leq f(n_{i+1})$ since n is expanded before n_{i+1} .
 * Contradiction from b&c!

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Consequences of monotonicity

4. With a monotone heuristic, the first time A^* expands a state, it has found the minimum cost path to that state.
 - Proof:
 - * Let $\text{PATH1} = \langle \text{Start}, n_0, n_1, \dots, n_k, n \rangle$ be the first path to n found. We have $f(\text{path1}) = c(\text{PATH1}) + h(n)$.
 - * Let $\text{PATH2} = \langle \text{Start}, m_0, m_1, \dots, m_j, n \rangle$ be another path to n found later. we have $f(\text{path2}) = c(\text{PATH2}) + h(n)$.
 - * By property (3), $f(\text{path1}) \leq f(\text{path2})$
 - * hence: $c(\text{PATH1}) \leq c(\text{PATH2})$

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Consequences of monotonicity

- Complete.
 - Yes, consider a least cost path to a goal node $\text{SolutionPath} = \langle \text{Start} \rightarrow n_1 \rightarrow \dots \rightarrow G \rangle$ with cost $c(\text{SolutionPath})$
 - Since each action has a cost $\geq \epsilon > 0$, there are only a finite number of nodes (paths) that have cost $\leq c(\text{SolutionPath})$.
 - All of these paths must be explored before any path of cost $> c(\text{SolutionPath})$.
 - So eventually SolutionPath , or some equal cost path to a goal must be expanded.
- Time and Space complexity.
 - When $h(n) = 0$, for all n
 - h is monotone.
 - A^* becomes uniform-cost search!
 - It can be shown that when $h(n) > 0$ for some n , the number of nodes expanded can be no larger than uniform-cost.
 - Hence the same bounds as uniform-cost apply. (These are worst case bounds).

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Consequences of monotonicity

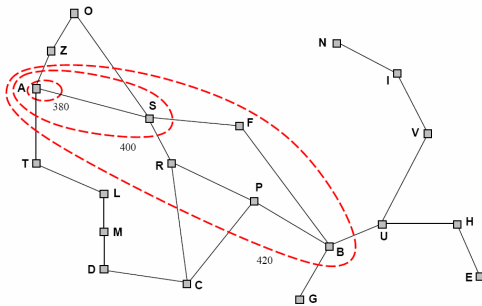
- Optimality
 - Yes, by (4) the first path to a goal node must be optimal.
- Cycle Checking
 - If we do cycle checking (e.g. using **GraphSearch** instead of **TreeSearch**) it is still optimal. Because by property (4) we need keep only the first path to a node, rejecting all subsequent paths.

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Search generated by monotonicity

Gradually adds “*f*-contours” of nodes (cf. breadth-first adds layers)
 Contour *i* has all nodes with $f = f_i$, where $f_i < f_{i+1}$



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Admissibility without monotonicity

- When “*h*” is admissible but not monotonic.
 - Time and Space complexity remain the same. Completeness holds.
 - Optimality still holds (without cycle checking), but need a different argument: don't know that paths are explored in order of cost.
- Proof of optimality (without cycle checking):
 - Assume the goal path $\langle S, \dots, G \rangle$ found by A^* has cost bigger than the optimal cost: i.e. $C^* < f(G)$.
 - There must exist a node *n* in the optimal path that is still in the frontier.
 - We have: $f(n) = g(n) + h(n) \leq g(n) + h^*(n) = C^* < f(G)$
 - Therefore, $f(n)$ must have been selected before G by A^* . contradiction!

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Admissibility without monotonicity

- No longer guaranteed we have found an optimal path to a node *the first time* we visit it.
- So, cycle checking might not preserve optimality.
 - To fix this: for previously visited nodes, must remember cost of previous path. If new path is cheaper must explore again.
- contours of monotonic heuristics don't hold.

Space problem with A^* (like breath-first search):

IDA* is similar to Iterative Lengthening Search: It puts the newly expanded nodes in the **front** of frontier! Two new parameters:

- **curBound** (any node with a bigger *f* value is discarded)
- **smallestNotExplored** (the smallest *f* value for discarded nodes in a round) when frontier becomes empty, the search starts a new round with this bound.

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Building Heuristics: Relaxed Problem

- One useful technique is to consider an easier problem, and let $h(n)$ be the cost of reaching the goal in the easier problem.
- 8-Puzzle moves.
 - Can move a tile from square A to B if
 - A is adjacent (left, right, above, below) to B
 - and B is blank
- Can relax some of these conditions
 1. can move from A to B if A is adjacent to B (ignore whether or not position is blank)
 2. can move from A to B if B is blank (ignore adjacency)
 3. can move from A to B (ignore both conditions).

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Building Heuristics: Relaxed Problem

- #3 leads to the **misplaced tiles** heuristic.
 - To solve the puzzle, we need to move each tile into its final position.
 - Number of moves = number of misplaced tiles.
 - Clearly $h(n)$ = number of misplaced tiles \leq the $h^*(n)$ the cost of an optimal sequence of moves from n .
- #1 leads to the **manhattan distance** heuristic.
 - To solve the puzzle we need to slide each tile into its final position.
 - We can move vertically or horizontally.
 - Number of moves = sum over all of the tiles of the number of vertical and horizontal slides we need to move that tile into place.
 - Again $h(n)$ = sum of the manhattan distances $\leq h^*(n)$
 - in a real solution we need to move each tile at least that far and we can only move one tile at a time.

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Building Heuristics: Relaxed Problem

- The optimal cost to nodes in the relaxed problem is an **admissible heuristic** for the original problem!
Proof: the optimal solution in the original problem is a (*not necessarily optimal*) solution for relaxed problem, therefore it must be at least as expensive as the optimal solution in the relaxed problem.
- Comparison of IDS and A* (average total nodes expanded):

Depth	IDS	A*(Misplaced)	A*(Manhattan)
10	47,127	93	39
14	3,473,941	539	113
24	---	39,135	1,641

Let h_1 =Misplaced, h_2 =Manhattan

- Does h_2 **always** expand less nodes than h_1 ?
 - Yes! Note that h_2 dominates h_1 , i.e. for all $n: h_1(n) \leq h_2(n)$. From this you can prove h_2 is faster than h_1 .
 - Therefore, among several admissible heuristic the one with highest value is the fastest.

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Building Heuristics: Pattern databases.

- Admissible heuristics can also be derived from solution to **subproblems**: Each state is mapped into a **partial specification**, e.g. in 15-puzzle only **position of specific tiles matters**.

- Here are goals for two sub-problems (called Corner and Fringe) of 15puzzle.

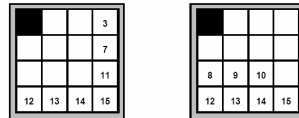


Fig. 2. The Fringe and Corner Target Patterns.

- By searching **backwards** from these goal states, we can compute the distance of any configuration of these tiles to their goal locations. We are ignoring the identity of the other tiles.
- For any state n , the number of moves required to get these tiles into place form a lower bound on the cost of getting to the goal from n .

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Building Heuristics: Pattern databases.

- These configurations are stored in a database, along with the number of moves required to move the tiles into place.
- The **maximum** number of moves taken **over all of the databases** can be used as a heuristic.
- On the 15-puzzle
 - The fringe data base yields about a 345 fold decrease in the search tree size.
 - The corner data base yields about 437 fold decrease.
- Some times **disjoint patterns** can be found, then the number of moves can be **added** rather than taking the max.

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Local Search

- So far, we keep the paths to the goal.
- For some problems (like 8-queens) we don't care about the path, we only care about the solution. Many real problem like Scheduling, IC design, and network optimizations are of this form.
- **Local search** algorithms operate using a single **Current state** and generally move to neighbors of that state.
- There is an **objective function** that tells the value of each state. The goal has the highest value (global maximum).
- Algorithms like **Hill Climbing** try to move to a neighbor with the highest value.
- Danger of being stuck in a **local maximum**. So some randomness can be added to "shake" out of local maxima.

Local Search

- **Simulated Annealing**: Instead of the best move, take a random move and if it improves the situation then always accept, otherwise accept with a probability < 1 . Progressively decrease the probability of accepting such moves.
- **Local Beam Search** is like a parallel version of Hill Climbing. Keeps K states and at each iteration chooses the K best neighbors (so information is shared between the parallel threads). Also stochastic version.
- **Genetic Algorithms** are similar to Stochastic Local Beam Search, but mainly use **crossover** operation to generate new nodes. This swaps feature values between 2 parent nodes to obtain children. This gives a hierarchical flavor to the search: chunks of solutions get combined. Choice of state representation becomes very important. Has had wide impact, but not clear if/when better than other approaches.