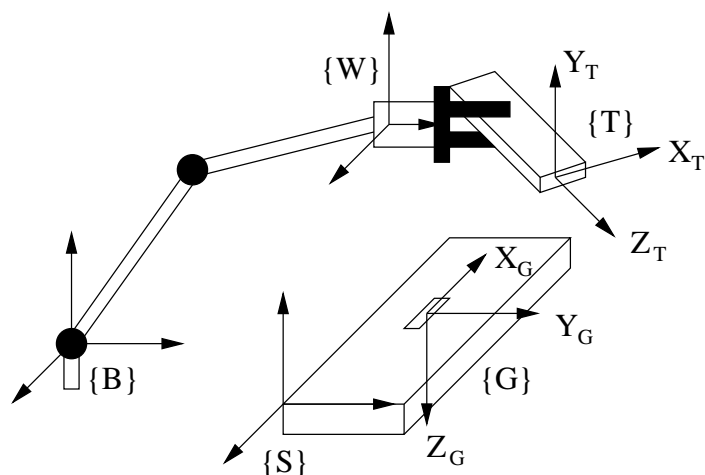


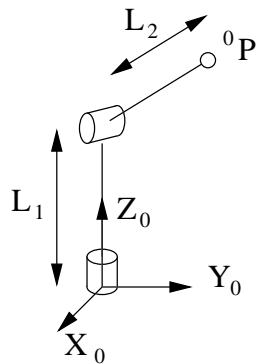
1. Show that the distance between points is unchanged by rotation; that is $\|p_1 - p_2\| = \|Rp_1 - Rp_2\|$.
2. Suppose that A is a 2×2 matrix where $A^T A = I$ and $\det A = 1$. Show that there exists a unique θ such that

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

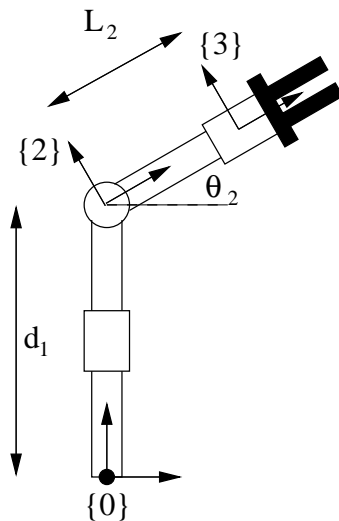
3. (a) Consider all pairs of rotations and translations along the three principle axes, i.e., $R_x R_x, R_x R_y, \dots, R_x T_x, \dots, T_z T_z$. Which pairs commute?
 (b) Given your answer to (a), what other representations are there for the Denavit-Hartenberg transformation?
4. (a) Suppose you have a frame $\{A\}$ and a frame $\{B\}$. The 4×4 homogeneous matrix T_B^A , where the upper-left 3×3 sub-matrix is a rotation matrix, has three distinct interpretations. What are the interpretations?
 (b) Consider the figure shown below. The pose of the tool relative to the wrist, T_T^W , is not known. By limping the arm joints, the tool tip can be inserted into the socket, or goal, at location T_G^S . In this calibration configuration, frames $\{G\}$ and $\{T\}$ are coincident, and the pose of the wrist relative to the base, T_W^B , can be retrieved from the robot. Assuming T_S^B and T_G^S are known, give the transform equation to compute the unknown pose of the tool, T_T^W .



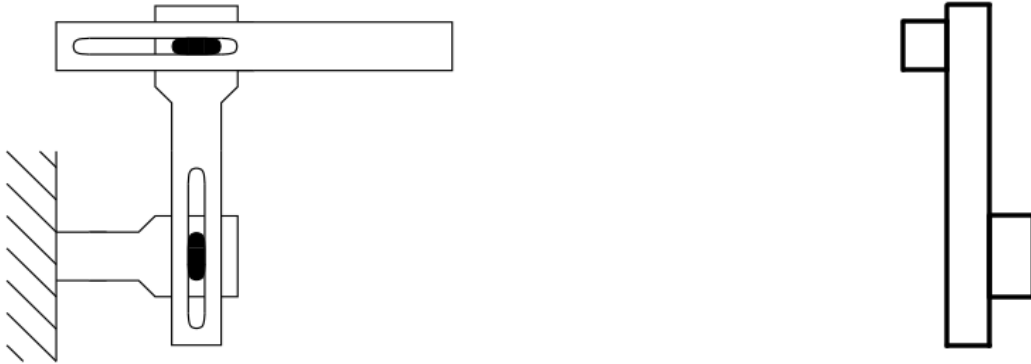
5. (a) What is the forward kinematics problem for a robotic arm?
- (b) What is the inverse kinematics problem for a robotic arm?
- (c) Consider the RR robot (shown below), that is similar to a robot made up of the waist and shoulder joints of the A150 robot. Given a point ${}^0P = [x \ y \ z]^T$ known to be in the workspace of the robot what are the joint angles θ_1 and θ_2 ? Assume that θ_1 is measured from X_0 and θ_2 is measured from the horizon.



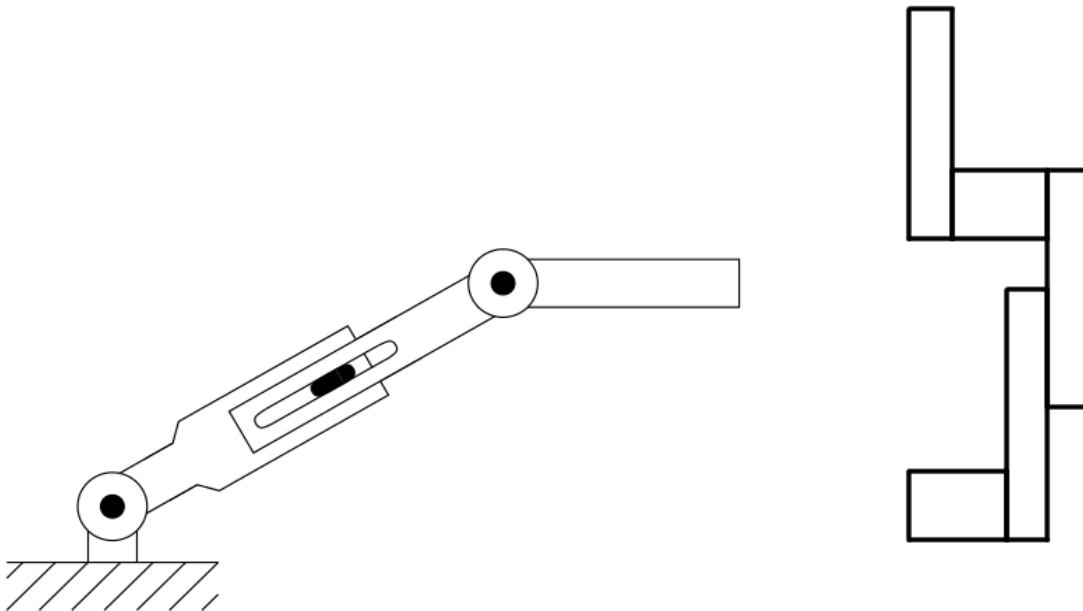
- (d) Consider the robotic arm shown below made up of a prismatic joint (moving vertically) and a revolute joint (positive rotation counter-clockwise in the page). Derive the matrix T_3^0 . Assume that the frames shown indicate the X and Y axes of the frames. Do not use the Denavit-Hartenberg convention to obtain a solution; the manipulator is simple enough that you should be able to derive a solution using basic linear algebra.



6. (a) Consider the figure shown below of a PP robot (left: view of the side of the robot, right: view of the front of the robot). Derive the forward kinematics of the robot using the DH-convention; choose your own variables for the missing dimensions.

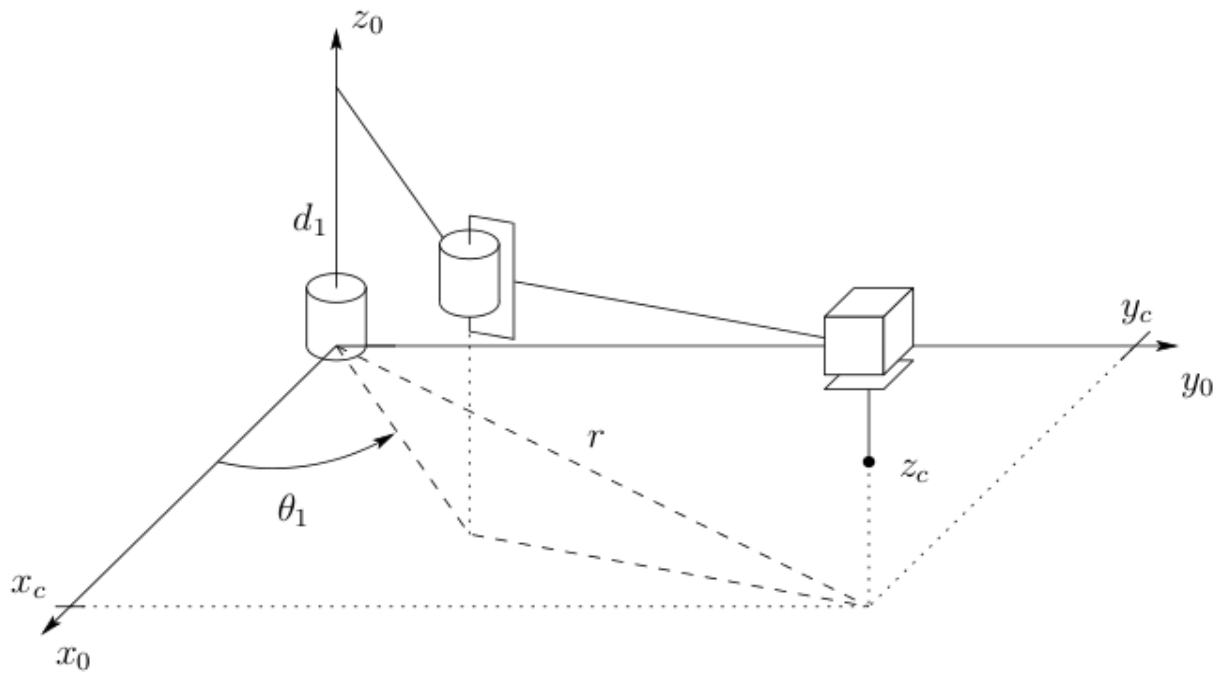


- (b) Given the end effector position (say at the end of link 3), solve for the inverse kinematics of the robot.
7. (a) Consider the figure shown below of a RPR robot (left: view of the side of the robot, right: view of the top of the robot). Derive the forward kinematics of the robot using the DH-convention; choose your own variables for the missing dimensions.



- (b) Given the end effector position (say at the end of link 4), solve for the inverse kinematics of the robot.

8. (a) Consider the figure shown below of a SCARA robot. Derive the forward kinematics of the robot using the DH-convention; choose your own variables for the missing dimensions.



- (b) Given the wrist center location $[x_c \ y_c \ z_c]^T$, solve for the inverse kinematics of the robot.