EECS4421Z: Introduction to Robotics

Midterm

Instructor: Dr. Burton Ma

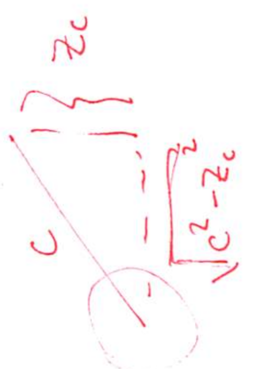
Thu Feb 15, 2018

Name: ______________________________

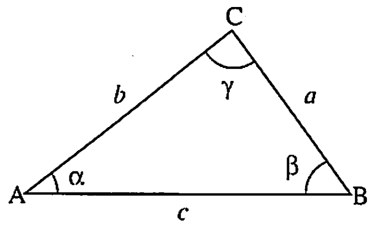
Student Number: ______________________________

Instructions

1. You have 110 minutes to complete the exam.
2. Write your answers clearly and succintly in the space provided on the question sheets; use the back of the page and the extra blank page if you need additional space for your answer.
3. A non-programmable calculator is allowed but should not be necessary. No other aids are allowed.



Law of cosines:



$c^2 = a^2 + b^2 - 2ab\cos\gamma$

Trigonometric identities:

$\sin(-\theta) = -\sin(\theta)$
$\cos(-\theta) = \cos(\theta)$

$\sin(180 - \theta) = \sin(\theta)$
$\cos(180 - \theta) = -\cos(\theta)$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos^2\theta + \sin^2\theta = 1$

Canonical rotation matrices:

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation about a unit axis $[k_x\ k_y\ k_z]^T$:

$$R_{k,\theta} = \begin{bmatrix} k_x^2 v_\theta + c_\theta & k_x k_y v_\theta - k_z s_\theta & k_x k_z v_\theta + k_y s_\theta \\ k_x k_y v_\theta + k_z s_\theta & k_y^2 v_\theta + c_\theta & k_y k_z v_\theta - k_x s_\theta \\ k_x k_z v_\theta - k_y s_\theta & k_y k_z v_\theta + k_x s_\theta & k_z^2 v_\theta + c_\theta \end{bmatrix}$$

where $c_\theta = \cos\theta$, $s_\theta = \sin\theta$, and $v_\theta = 1 - \cos\theta$.

Homogeneous translation matrix for a translation of $[x\ y\ z]^T$:

$$D = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

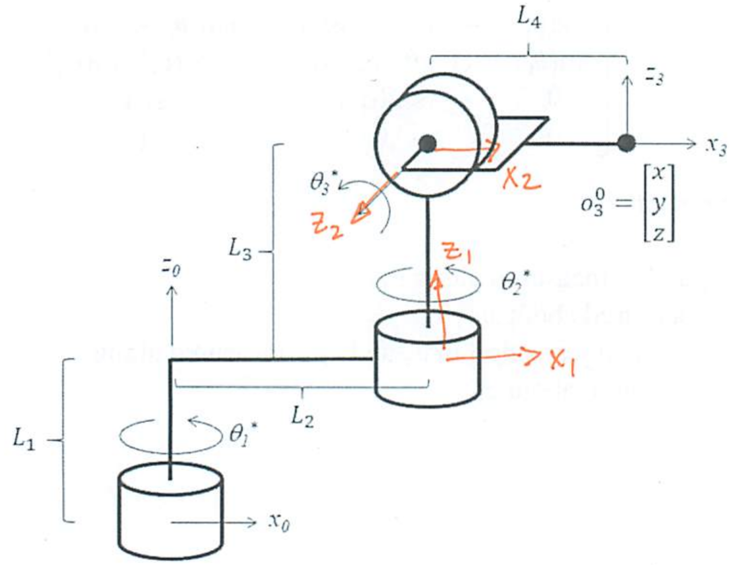
Denavit-Hartenberg transformation:

$$
\begin{aligned}
T_i^{i-1} &= R_z(\theta_i) D_z(d_i) D_x(a_i) R_x(\alpha_i) \\
&= \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Denavit-Hartenberg parameters:

a_i: distance between z_{i-1} and z_i measured along x_i
α_i: angle from z_{i-1} to z_i measured about x_i
d_i: distance between o_{i-1} to the intersection of x_i and z_{i-1} measured along z_{i-1}
θ_i: angle from x_{i-1} to x_i measured about z_{i-1}

1. Consider the following RRR robot:



The robot is shown in a position where all of the joint angles are zero degrees. For the purposes of this question, assume that there are no limits on the joint angles.

(a) (2 points) Draw coordinate frames {1} and {2} that are suitable for use with the Denavit-Hartenberg convention.

(b) (6 points) Fill in the Denavit-Hartenberg parameters using your coodinate frame placement.

Link	a_i	α_i	d_i	θ_i
1	L_2 ($-L_2$ if $X_1 \leftarrow$)	0	L_1	θ_1^* ((θ_1^*+180) if $X_1 \leftarrow$)
2	0	90° ($-90°$ possible)	L_3 ($-L_3$ if $z_1 \downarrow$)	θ_2^* (θ_2^*+180 possible)
3	L_4	$-90°$ ($+90°$ possible)	0	θ_3^* ($180-\theta_3^*$ if $X_2 \leftarrow$)

(c) (12 points) Solve for the forward kinematics of the arm; that is, given joint angles θ_1^*, θ_2^*, and θ_3^*, find the orientation and position of frame {3} as a homogeneous matrix T_3^0. If your solution involves a sequence of matrix multiplications then you do not need to perform the actual multiplications. Do not use the Denavit-Hartenberg convention to obtain your answer.

Use this page for Question 1.

$$R_{z,\theta_1^*} D_{z,L_1} D_{x,L_2} R_{z,\theta_2^*} D_{z,L_3} R_{y,-\theta_3^*} D_{x,L_4}$$

2. Consider the RR arm shown in Figure 2.

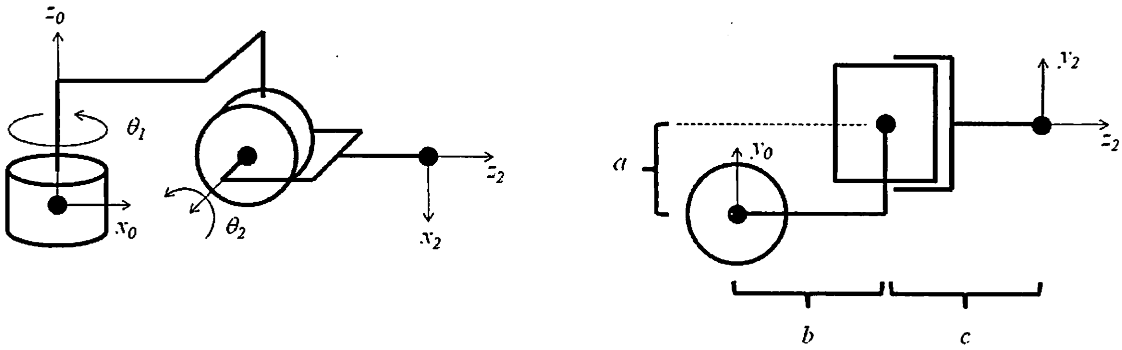


Figure 1: Left: Front view of arm. Right: Top-down view of arm. In this figure, all joint angles are shown at 0°.

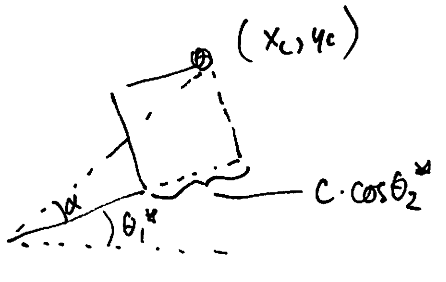
Joints 1 and 2 are connected by a link with a 90° bend. Joint 1 has a range of $-360 < \theta_1^* < 360$ degrees. The axis of joint 2 is always in the plane $z_0 = 0$. You may assume that the link dimensions a, b, and c are always greater than zero, and that $b > c$.

(a) (15 points) Assume that joint 2 has a range of $-90 <= \theta_2^* <= 90$ degrees. Given $o_c = [x_c\ y_c\ z_c]^T$, the location of the origin of frame $\{2\}$ relative to frame $\{0\}$, solve for the inverse kinematics of arm.

(b) (10 points) Assume that joint 2 has a range of $-180 <= \theta_2^* <= 180$ degrees. How do you have to modify your answer to (a) to solve for the inverse kinematics of the arm?

$\theta_2^* = \operatorname{asin}\left(\frac{z_c}{c}\right)$ – must compute this first

looking top down



$\alpha = \operatorname{atan2}(a,\ b + c\cos\theta_2^*)$

$\theta_1^* = \operatorname{atan2}(y_c, x_c) - \alpha$

Use this page for Question 2 if necessary.

(b) Compute $r = \sqrt{x_c^2 + y_c^2}$

if $r \geq \sqrt{a^2 + b^2}$ then $-90 \leq \theta_2^* \leq 90$ and solution (a) applies

if $r < \sqrt{a^2 + b^2}$

$$\cancel{\theta_2^* = \operatorname{asin}\left(\frac{z_c}{c}\right)}$$

$$\theta_2^* = \begin{cases} 180 - \operatorname{asin}\left(\frac{z_c}{c}\right) & \text{if } z_c > 0 \\ -180 - \operatorname{asin}\left(\frac{z_c}{c}\right) & \text{if } z_c < 0 \end{cases}$$

θ_1^* can be computed as in (a)

3. Given a rotation matrix R:

   (a) (3 points) How can you find the axis of rotation of R?

   (b) (2 points) How can you find the angle of rotation of R?

   For your answers to (a) and (b), you may provide a formula or a description of the computation that you would have to perform.

3(a) Rotation about an axis leaves all points on the axis unchanged; i.e.

$$R \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} = 1 \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

This is an eigenvalue problem - find the eigenvector of R whose eigenvalue is 1; the eigenvector is $\begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$

3(a) Examine $R_{k,\theta} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ (see Equations on pg 1)

$$r_{32} - r_{23} = 2 k_x \sin\theta$$
$$r_{13} - r_{31} = 2 k_y \sin\theta$$
$$r_{21} - r_{12} = 2 k_z \sin\theta$$

$$\therefore \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} = \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} \Bigg/ \left\| \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} \right\|$$

3(b) Examine $R_{k,\theta}$

$$\begin{aligned} r_{11} + r_{22} + r_{33} &= k_x^2 V_\theta + C_\theta + k_y^2 V_\theta + C_\theta + k_z^2 V_\theta + C_\theta \\ &= (k_x^2 + k_y^2 + k_z^2) V_\theta + 3C_\theta \\ &= 1(1 - C_\theta) + 3C_\theta \\ &= 1 + 2\cos\theta \end{aligned}$$

$$\therefore \theta = \operatorname{acos}\left(\frac{r_{11} + r_{22} + r_{33} - 1}{2} \right)$$