EECS4421Z/5324M: Lab 2

Thu Jan 25, 2018
Due: In class Wed Feb 7, 2017

1 A150 Forward and Partial Inverse Kinematics

Note: I will check your answers for steps 1 and 3 if you ask.

1. Derive the table of Denavit-Hartenberg (DH) parameters for the A150 robot using the frame placements shown in Figure 1. Links 1–3 all have a length of 10 inches. Link 4 can be treated as a link of length 0 inches. The distance between o_4 and o_5 is 2 inches.

a	α	d	θ
0	90	10	θ_1^*
10	0	0	θ_2^*
10	0	0	θ_3^*
0	90	0	$\theta_4^* + 90$
0	0	2	$\theta_5^* + 180$

2. Implement a Matlab function that computes the Denavit-Hartenberg transformation matrix given vectors of DH values a, α, d, and θ. The function signature should be:

```
function T = dh(a, alpha, d, theta)
```

For example, if `a`, `alpha`, `d`, and `theta` were all vectors of length 5 then `T = dh(a, alpha, d, theta)` would compute the matrix $T = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4$ where the T_j^i are Denavit-Hartenberg transformation matrices. Your function should work for any robot described by a, α, d, and θ (not just the A150 robot).

You can check that your function gives results that are consistent with the A150 simulator by plugging in DH parameter values for the A150 arm and asking the simulator for the arm pose (use the `getpose` function of the simulator).

3. Derive the analytic form of the matrix T_5^3; i.e., derive the elements of the 4×4 matrix.

$$\begin{pmatrix} -\cos(\theta_5^*)\cos\left(\theta_4^* + \frac{\pi}{2}\right) & \cos\left(\theta_4^* + \frac{\pi}{2}\right)\sin(\theta_5^*) & \sin\left(\theta_4^* + \frac{\pi}{2}\right) & 2\sin\left(\theta_4^* + \frac{\pi}{2}\right) \\ -\cos(\theta_5^*)\sin\left(\theta_4^* + \frac{\pi}{2}\right) & \sin(\theta_5^*)\sin\left(\theta_4^* + \frac{\pi}{2}\right) & -\cos\left(\theta_4^* + \frac{\pi}{2}\right) & -2\cos\left(\theta_4^* + \frac{\pi}{2}\right) \\ -\sin(\theta_5^*) & -\cos(\theta_5^*) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4. Solve the inverse kinematics problem for the wrist; i.e., given T_5^3 solve for the values of θ_4 and θ_5.

There are several possible solutions; below is one of the solutions:

$$\begin{aligned} \theta_4 &= \text{atan2}(r_{13}, -r_{23}) \\ \theta_5 &= \text{atan2}(-r_{31}, -r_{32}) + 180 \end{aligned}$$

where r_{ij} is the element at row i and column j of the matrix in 3.

Also acceptable would be:

$$\begin{aligned}\theta_4^* &= \text{atan2}(r_{13}, -r_{23}) - \frac{\pi}{2} \\ \theta_5^* &= \text{atan2}(-r_{31}, -r_{32})\end{aligned}$$

5. Implement a Matlab function that computes the inverse kinematics of the wrist. The function signature should be:

```
function theta45 = invwrist(T35)
```

   where `theta45` is the vector $[\theta_4 \ \ \theta_5]$ and `T35` is the matrix T_5^3.

6. Implement a Matlab function that finds the location of o_c^0, the wrist center relative to frame $\{0\}$, given T_5^0, the pose of frame $\{5\}$ relative to frame $\{0\}$. The function signature should be:

```
function oc = wristcenter(T05)
```

   where `oc` is the wrist center location o_c^0 and `T05` is the matrix T_5^0.

7. Implement a Matlab function that solves the inverse position kinematics of the first three joints of the A150 arm. Given T_5^0, your function should return the three joint angles θ_1, θ_2, θ_3. The function signature should be:

```
function oc = invposkin(T05)
```

   where `T05` is the matrix T_5^0. You will need to use your function `wristcenter` from the previous part to obtain the wrist center position.

Submit your Matlab files using the command

```
submit 4421 lab2 dh.m invwrist.m wristcenter.m invposkin.m
```

Submit your written answers to 1, 3, and 4 along with your answers to the additional written questions found at the end of this document.

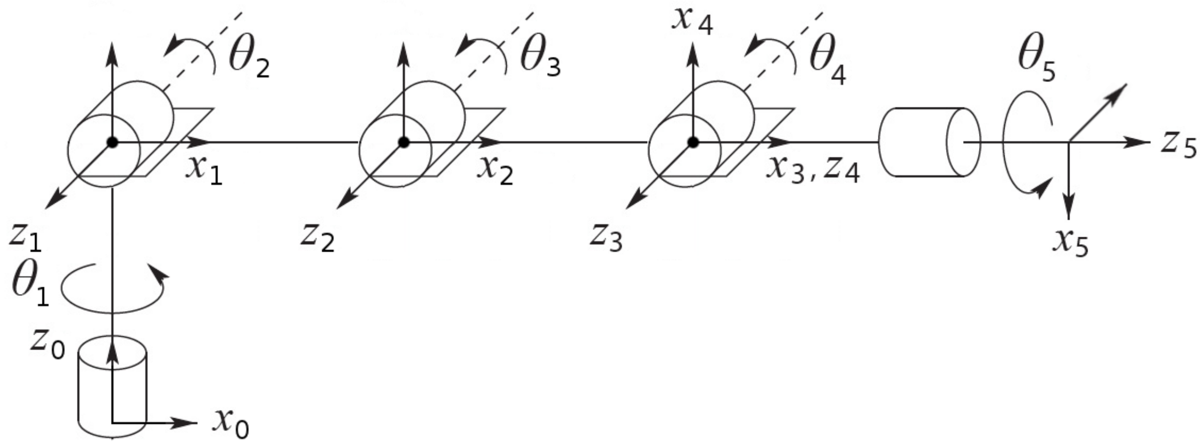

Figure 1: Denavit-Hartenberg frame placement for the A150 and A255 robots.

Joint variable	Range
θ_1	$-175°$ to $175°$
θ_2	$0°$ to $110°$
θ_3	$-130°$ to $0°$
θ_4	$-110°$ to $110°$
θ_5	$-180°$ to $180°$

Table 1: The joint variable ranges in the Denavit-Hartenberg convention.

2 Written Questions (Forward Kinematics)

1. Consider the RR arm shown in the figure below where joint 2 is always in the same plane as o_0.

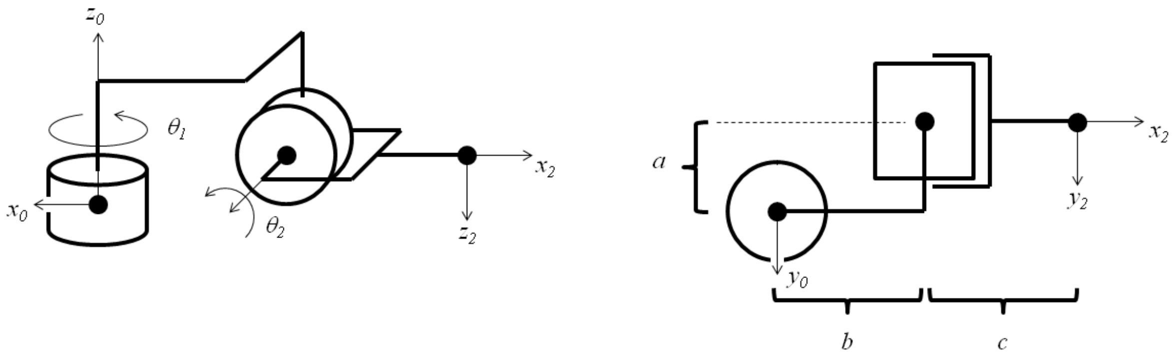

Figure 2: Left: Front view of arm. Right: Top-down view of arm. In this figure, all joint angles are shown at 0°.

Without using the Denavit-Hartenberg convention, give an expression for all 16 elements of T_2^0 when the robot has joint angles θ_1 and θ_2. Please show your work; if you use an intermediary frame (and you should), draw a figure showing the placement of the frame on the robot.

There are many possible ways to compute the solution, but you should end up with the same matrix regardless of how you compute it:

$$
\begin{aligned}
T_2^0 &= R_{z,\theta_1} R_{y,180} D_{x,b} D_{y,-a} R_{y,\theta_2} D_{x,c} \\
&= \begin{pmatrix} -\cos(\theta_1)\,\cos(\theta_2) & -\sin(\theta_1) & -\cos(\theta_1)\,\sin(\theta_2) & a\,\sin(\theta_1) - b\,\cos(\theta_1) - c\,\cos(\theta_1)\,\cos(\theta_2) \\ -\cos(\theta_2)\,\sin(\theta_1) & \cos(\theta_1) & -\sin(\theta_1)\,\sin(\theta_2) & -a\,\cos(\theta_1) - b\,\sin(\theta_1) - c\,\cos(\theta_2)\,\sin(\theta_1) \\ \sin(\theta_2) & 0 & -\cos(\theta_2) & c\,\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}
$$

2. Using the Denavit-Hartenberg convention, provide a figure showing the placement of frame 1 for the robot in Question 1. Provide the table of Denavit-Hartenberg parameters and compute the resulting Denavit-Hartenberg transformation matrix T_2^0 (again, showing the values of all 16 elements). It should be the same as your answer for Question 1.

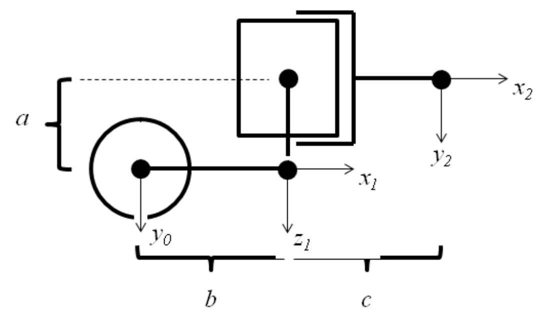

Figure 3: DH frame placement

a	α	d	θ
b	90	0	$\theta_1^* + 180$
c	90	$-a$	θ_2^*

$$T_2^0 = \begin{pmatrix} -\cos(\theta_1)\cos(\theta_2) & -\sin(\theta_1) & -\cos(\theta_1)\sin(\theta_2) & a\sin(\theta_1) - b\cos(\theta_1) - c\cos(\theta_1)\cos(\theta_2) \\ -\cos(\theta_2)\sin(\theta_1) & \cos(\theta_1) & -\sin(\theta_1)\sin(\theta_2) & -a\cos(\theta_1) - b\sin(\theta_1) - c\cos(\theta_2)\sin(\theta_1) \\ \sin(\theta_2) & 0 & -\cos(\theta_2) & c\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. Consider the robot shown in Figure 4.

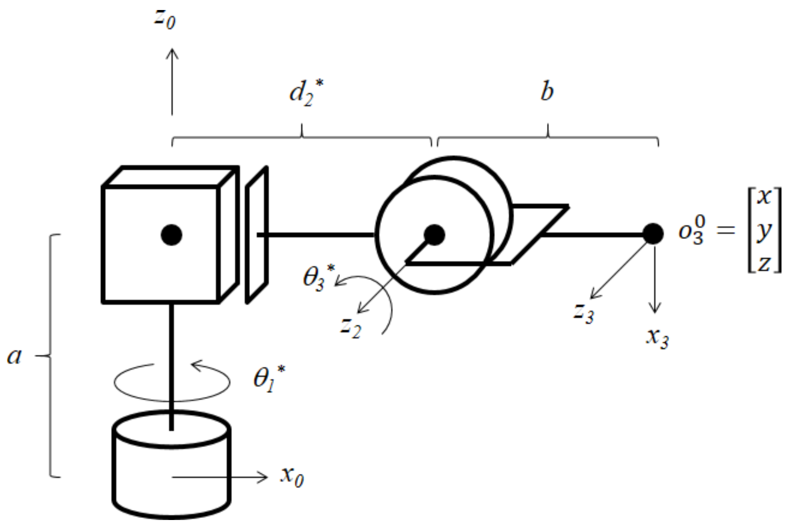

Figure 4: An RPR robot. The figure shows the robot with joint angles $\theta_1 = 0$ and $\theta_3 = 0$.

Redraw Figure 4 to include frame $\{1\}$ and frame $\{2\}$ placed according to the Denavit-Hartenberg convention, and provide the table of Denavit-Hartenberg parameters.

Note: There is an error in the direction of x_3 in the above figure; the intended placement is shown below.

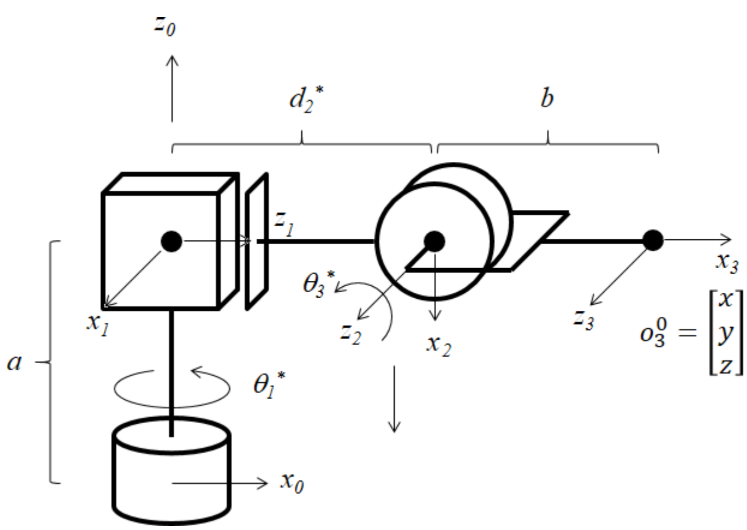

Figure 5: DH frame placement

a	α	d	θ
0	-90	a	$\theta_1^* - 90$
0	90	d_2^*	90
b	0	0	$\theta_3^* + 90$