Homework Assignment #9A
Due: August 21, 2018 at 4:00 p.m.

Submission instructions: Drop off a hard copy of your solution at EECS undergraduate office (and ask that it be put in my mailbox), or send me a pdf version by email.

1. Suppose you have computed the matrix $A[1..n, 1..n]$ of distances between all pairs of nodes in an n-node undirected graph G with weighted edges. Now, suppose one more edge (a, b) with weight w is added to G to produce a new graph G'. Give an algorithm to compute the distance matrix $B[1..n, 1..n]$ for G' in $O(n^2)$ time.

2. Once upon a time, people used paper maps instead of Google maps and GPS. These maps often had a matrix showing the shortest distances by road between pairs of cities. For an example, see http://www.mto.gov.on.ca/english/traveller/map/Triangle.pdf .

Archeologists have found a portion of a map of ancient Leutonia. Unfortunately, the only legible portion of it is the distance matrix, which gives the distances between all pairs of Leutonian cities; the rest of the map has turned to dust. The archeologists would like to reconstruct the network of roads that connected the Leutonian cities in ancient times. As is well-known, Leutonians were extremely superstitious and believed that two roads should only intersect at a temple dedicated to Zobnab, the god of traffic jams. Temples can only be built in cities, so no roads can intersect anywhere outside a city. The ancient Leutonians were famously lazy, so they wouldn't have built more roads than necessary. Thus, the archeologists want to find the *minimal* set of roads connecting pairs of cities that could explain the distances given in the matrix. The legend at the bottom of the distance matrix says that the distances are always measured from the temple of Zobnab in one city to the temple of Zobnab in the other.

(a) We can number the cities from 1 to n. Let $C[i, j]$ be the distance from city i to city j. Give an efficient algorithm that takes matrix C as input and reconstructs a weighted, undirected graph whose nodes represent the Leutonian cities and whose edges represent roads between cities. The weights on the edges should be the length of the road between the two cities (as measured between temples of Zobnab).

(b) Explain why your algorithm is correct. You should explain why the distances between nodes in the graph G will be given by the entries in C and why no graph with fewer edges can have this property.

(c) Give the worst-case running time of your algorithm as a function of n using Θ notation.