# Homework Assignment #6
# Due: March 9, 2018 at 2:30 p.m.

**1.** With a looming strike, escalation of nuclear war rhetoric and the threat of cataclysmic global warming, we could all use a little hygge, which is a Danish word for a feeling of cozy contentment. In practice, this seems to involve lots of fuzzy blankets, candles and tea. A certain purveyor of cheap furniture and accessories wants to take advantage of the hygge fad by selling more candles and shelves to put them on. They have designed a new shelf that has space to hold up to 100 tealights. Before they start to sell the shelves, they want to test them to see how exactly how many burning tealights can be placed on a shelf before the shelf itself catches fire. When they calculate the maximum number $m$ that can be safely burned simultaneously (i.e., without causing the shelf itself to ignite), they will attach stickers to the shelves that say "Warning: do not place more than $m$ burning tea lights on this shelf". A tealight burns for exactly 1 hour. Just to be safe from lawsuits, the company wishes to ensure that burning $m$ tea lights on the shelf for a full hour will not cause the shelf to burst into flames.

**(a)** Assume you have an unlimited supply of tealights. The shelf manufacturer gives you a single shelf to test. If, during your testing, the shelf bursts into flames, you cannot perform any further tests on that shelf. What should your strategy be for testing the shelf to determine $m$? How many tealights might you have to use for your testing in the worst case?

**(b)** Tealights aren't free, so you should try to use fewer of them during your testing if you can. Suppose the manufacturer gives you *two* sample shelves for your testing instead of one. Design a testing strategy whose worst-case number of tealights required is as small as possible. (Here, the worst case means the worst over all possible values that $m$ could have in the range 0 to 100.)

**(c)** Now suppose the capacity of a shelf is $k$ tealights (instead of 100) and you wish to find the maximum number $m$ (where $0 \leq m \leq k$) of tealights that can be burned simultaneously on the shelf without causing the shelf to catch fire. Design an algorithm that takes integers $k \geq 1$ and $n \geq 1$ as input and computes the smallest number of tealights that are needed (in the worst case) to perform the fire testing if you have access to $n$ sample shelves. You should make your algorithm as efficient as possible. Explain why your algorithm is correct.

**(d)** State the running time of your algorithm in part (c) in terms of $n$ and/or $k$ using $\Theta$ notation.