York University EECS 3101 January 26, 2018

Homework Assignment #3
Due: February 2, 2018 at 2:30 p.m.

1. Let $n \geq 1$. Suppose we have a collection of $N = 2^n$ computers. They are connected into a hypercube network as follows. Each computer is labelled by a binary string of length n. No two computers have the same label. There is a wire connecting two computers if and only if the two labels of the computers differ in exactly one bit. For example, if $n = 8$ there would be a direct connection between the computers labelled 01001001 and 01011001 because the two labels differ only in the value of their fourth bits. When $n = 3$, the network forms a cube as shown in the diagram below.

Suppose we want the computer with label $000\ldots0$ to send some information to all other computers, but we only want to transmit messages across $N - 1$ of the edges. Willemina figured out how to do this and wrote a recursive algorithm to print out the order in which the information should be passed around the network.

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List(n) : list of strings
    if n is 1 then output ⟨0, 1⟩
    else
        L ← List(n − 1)
        let x_1, x_2, ..., x_k be the elements of L (in order)
        output ⟨0x_1, 0x_2, ..., 0x_{k−1}, 0x_k, 1x_k, 1x_{k−1}, ..., 1x_2, 1x_1⟩
    end if
end List
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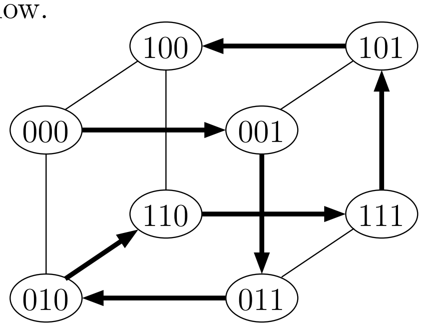
For example, List(3) outputs $\langle 000, 001, 011, 010, 110, 111, 101, 100\rangle$. Thus, according to Willemina's scheme, the message would be sent around the network as shown by the arrows below.



Prove that for all $n \geq 1$, the output of List(n) is correct. In other words, prove that it has the following two properties.

- Every binary string of length n is printed exactly once.
- If two labels appear next to each other in the list, then there is a wire between the two computers with those labels.