York University | EECS 3101 | January 12, 2018

Homework Assignment #1
Due: January 19, 2018 at 2:30 p.m.

When you submit your solution to Assignment 1, you should also hand in the declaration on academic honesty available on the course web page. Without this declaration, your assignment will not be marked. Please do not staple the declaration to your solutions.

1. In class, we proved that every natural number can be represented in binary. In fact, this proof was constructive: it essentially provides an algorithm to construct the binary representation of any given natural number. Here is that algorithm:

```
BINARY(n)
    ℓ ← 0
    r ← n
    loop
        exit when r = 0
        a_ℓ ← r mod 2
        r ← ⌊r/2⌋
        ℓ ← ℓ + 1
    end loop
    output a_{ℓ−1} a_{ℓ−2} ... a_2 a_1 a_0
    Postcondition: n = Σ_{i=0}^{ℓ−1} a_i · 2^i and for all i in the range 0 ≤ i ≤ ℓ − 1, a_i ∈ {0, 1}
end BINARY
```

(a) What is the precondition?

(b) It is fairly easy to show that r is always a natural number. Use this fact to show that the algorithm terminates whenever it is given an input that satisfies the precondition.

(c) State a loop invariant that is strong enough to prove the algorithm is correct. (In other words, you should be able to use it to do parts (d) and (e) below.)

(d) Prove the invariant in part (c) is indeed an invariant.

(e) Show that your invariant, together with the exit condition $r = 0$, implies the postcondition.