EECS 1028 M: Discrete Mathematics for Engineers

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Course page: `http://www.eecs.yorku.ca/course/1028`
Also on Moodle

A Simple Claim

Claim: Let $n \in \mathbb{N}$. If n^2 is even, then n is even.
Proof: Suppose this is false. Then n is odd. But $n^2 = n * n$ must be odd because the product of 2 odd numbers is odd. That is a contradiction!
Why? We assumed that n^2 is even but now it turns out that n^2 is odd.

Something went wrong! Our algebra was correct, so our original assumption (that n is odd) is incorrect.

Therefore n cannot be odd, or it must be even.

Note: There are several other ways to prove this.

Proof that $\sqrt{2}$ is not rational

Let's suppose the statement is false; i.e., $\sqrt{2}$ is a rational number. Then
$\sqrt{2} = a/b$ where a, b are integers, $b \neq 0$.
We ALSO assume that a/b is simplified to lowest terms, i.e., $gcd(a, b) = 1$. So,

$$\begin{aligned} \sqrt{2} &= a/b \text{ squaring} \\ 2 &= a^2/b^2, \text{ or} \\ a^2 &= 2b^2 \end{aligned}$$

So a^2 is even implying that a is also even.
Why? Because if a is odd, then a^2 is odd.
Since a is even, then $a = 2k$ for some integer k.

Proof that $\sqrt{2}$ is not rational - Continued

Substituting $a = 2k$ we get:

$$\begin{aligned} a^2 &= 2b^2 \\ 4k^2 &= 2b^2 \\ b^2 &= 2k^2 \end{aligned}$$

So b^2 is even, implying b is even. That is a contradiction.
Why? We assumed that $gcd(a, b) = 1$ but now it turns out that a, b are both even, so $gcd(a, b) \neq 1$.

So our original assumption (that $\sqrt{2}$ is rational) is incorrect.

Therefore $\sqrt{2}$ cannot be rational.