EECS 1028 M: Discrete Mathematics for Engineers

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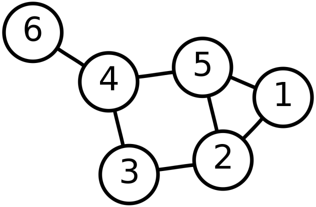
Course page: `http://www.eecs.yorku.ca/course/1028`
Also on Moodle

Graphs: Motivations and Basic Idea

Sec 10.1, 10.2

- Tool for modeling many real applications
- Abstract model that throws away many non-essential aspects of a problem
- Nodes, connected by edges



- No geographical locations attached to node positions, no significance of edge lengths

Graphs: Applications

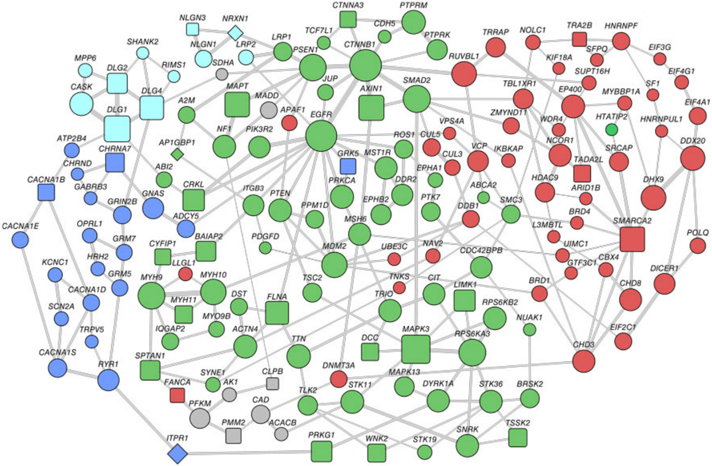
Many Applications, including:

- Road networks
- Subway/Train networks
- Airline networks
- Social Networks
- Power Grid
- Electronic Communication Networks
- Electrical Circuits
- Biological Networks
- Ecological Networks

Graphs: Applications - 2

- The web graph
- Software module dependencies
- Computation structure
- Scheduling constraints
- Collaboration graphs
- State graphs of machines and protocols
- Many, many others

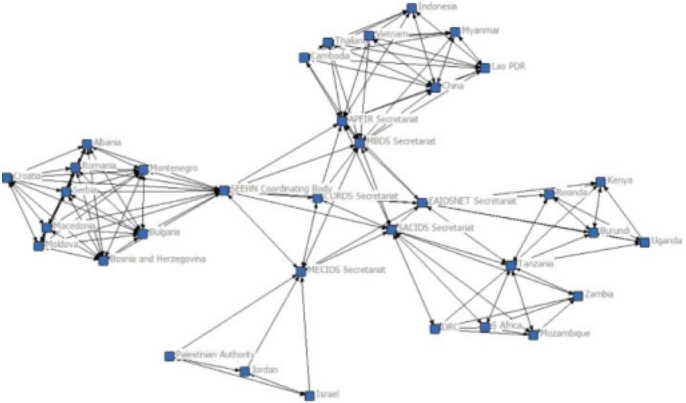
Graphs: Applications - 3



Article: Gene networks offer entry point to unraveling autism

From `https://spectrumnews.org/news/gene-networks-offer-entry-point-to-unraveling-autism/`

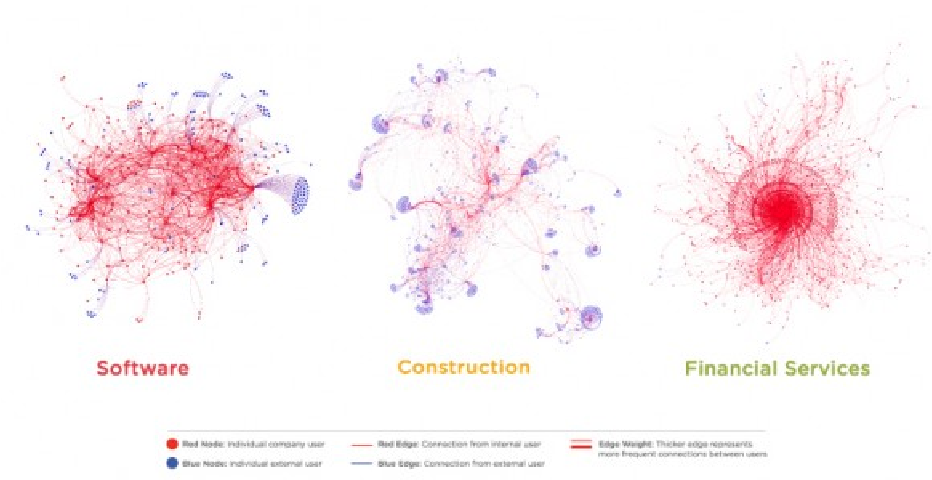
Graphs: Applications - 4



A social network graph illustrating the connections among countries and regional networks in CORDS (CORDS=Connecting Organizations for Regional Disease Surveillance;

From https://openi.nlm.nih.gov/detailedresult.php?img=PMC3557911_EHTJ-6-19913-g001&req=4

Graphs: Applications - 5



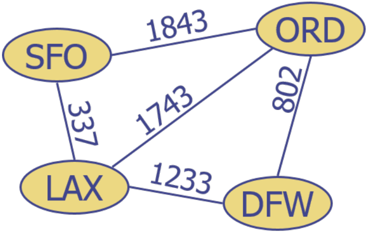
Collaboration graph among people in the same company

From `https://linkurio.us/blog/visualizing-business-organizations-the-collaboration-graph/`

Definitions

$G = (V, E)$, $V =$ set of nodes/vertices, $E =$ set of edges

- Edges incident on a vertex
- Adjacent vertices
- degree of a node
- neighborhood of a node
- Self-loops
- Edge weights



Definitions - 2

- Edge Types:
  - Directed edge: ordered pair of vertices (u, v)
    - u : origin, v : destination
  - Undirected edge: unordered pair of vertices (u, v)
- Graph Types:
  - Directed graph: all the edges are directed
  - Undirected graph: all the edges are undirected
- Paths:
  - Simple Paths
  - Cycles
  - Simple cycles: no vertex repeated

Graph Representations

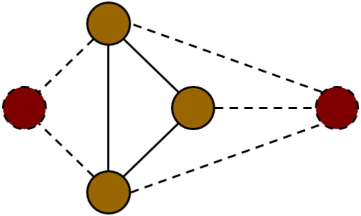
- Adjacency list
- Adjacency matrix
- Incidence matrix

Elementary Properties

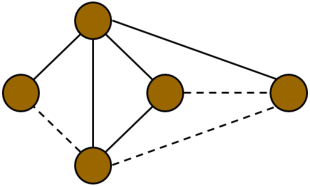
- **Handshaking Theorem**, Thm 1 (pg 653): Sum of degrees equals twice the number of edges in an undirected graph
- Thm 3 (pg 654) Sum of indegrees equals sum of outdegrees in a digraph, which in turn equals $|E|$
- In an undirected graph $m \leq \frac{n(n-1)}{2}$
  What is the bound for directed graphs?

Subgraphs

- A subgraph S of a graph G is a graph such that
  - The vertices of S are a subset of the vertices of G
  - The edges of S are a subset of the edges of G
- A spanning subgraph of G is a subgraph that contains all the vertices of G



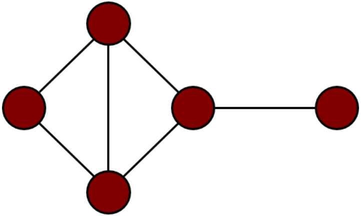
Subgraph



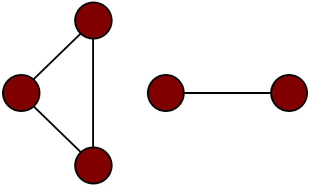
Spanning subgraph

Connected graphs

- A graph is connected if there is a path between every pair of vertices
- A connected component of a graph G is a maximal connected subgraph of G



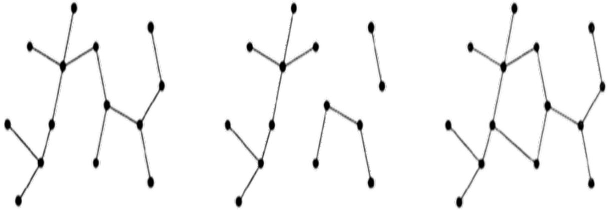
Connected graph



Disconnected graph with two connected components

Trees

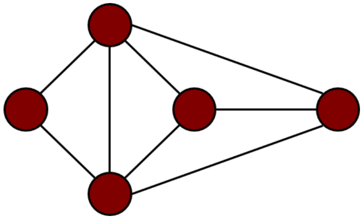
- A tree is a connected, acyclic, undirected graph
- A forest is a set of trees (not necessarily connected)



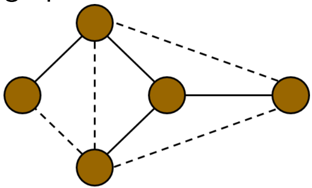
Tree, forest, a cyclic graph

Spanning Trees

- A spanning tree of a connected graph is a spanning subgraph that is a tree
- A spanning tree is not unique unless the graph is a tree
- Spanning trees have applications to the design of communication networks
- A spanning forest of a graph is a spanning subgraph that is a forest



graph



Spanning tree