Cellular Networks Modeling, Analysis, and Design Using Stochastic Geometry

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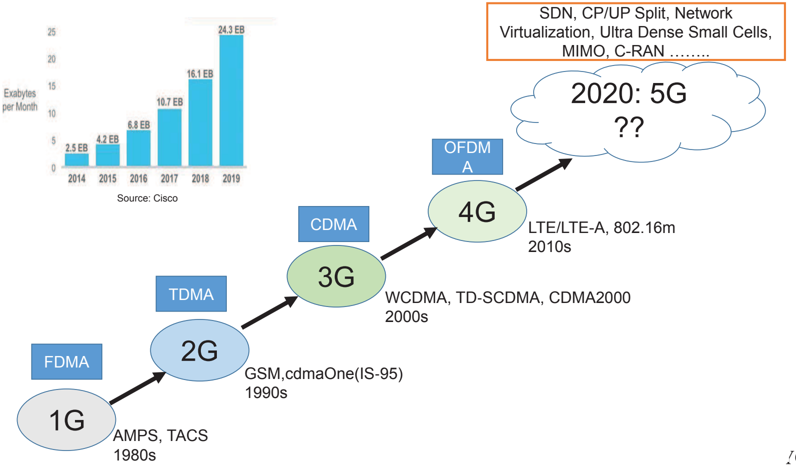
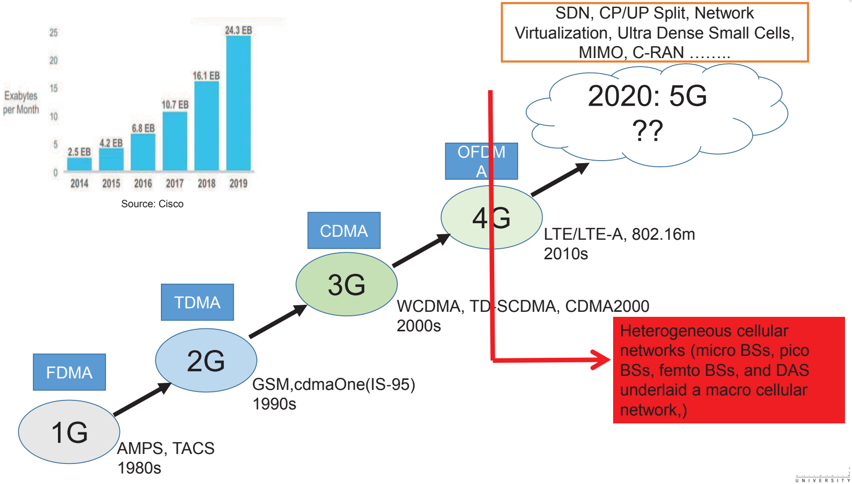
Outline

Roadmap to 5G



Source: Cisco

Roadmap to 5G



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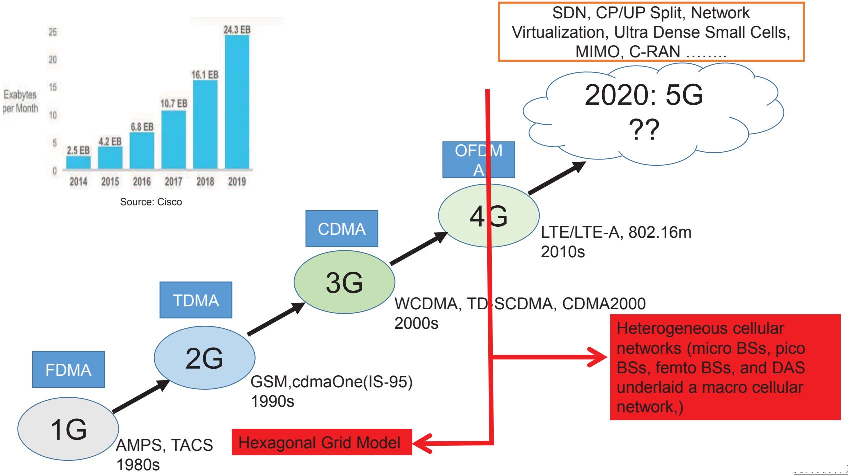

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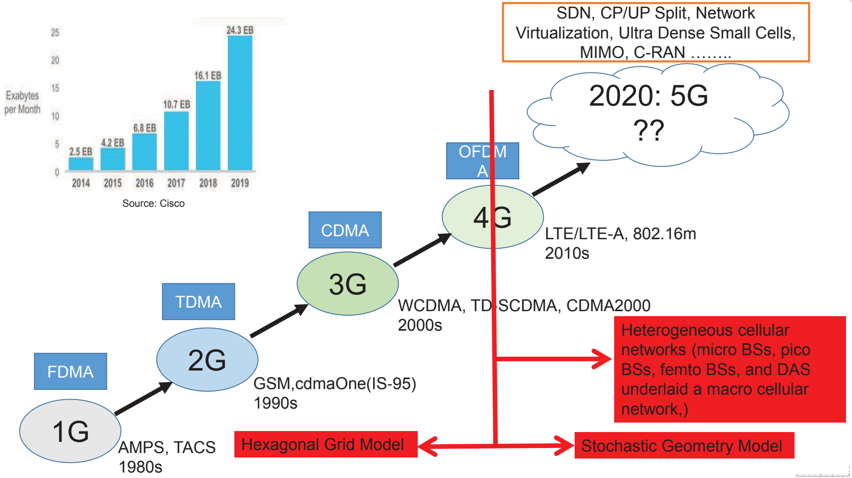
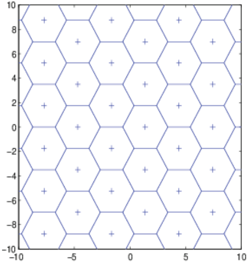
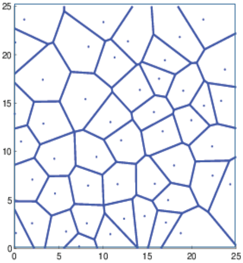
Roadmap to 5G



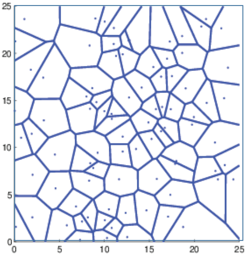
Modeling single-tier Cellular Network



Grid Model

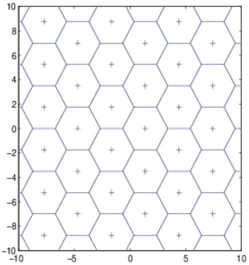


Actual 4G Deployment



Poisson Point Process (PPP)

Modeling single-tier Cellular Network



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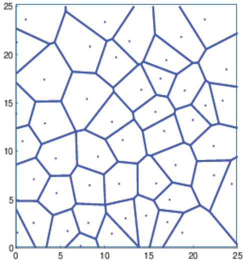
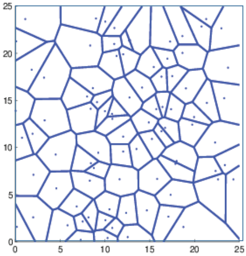


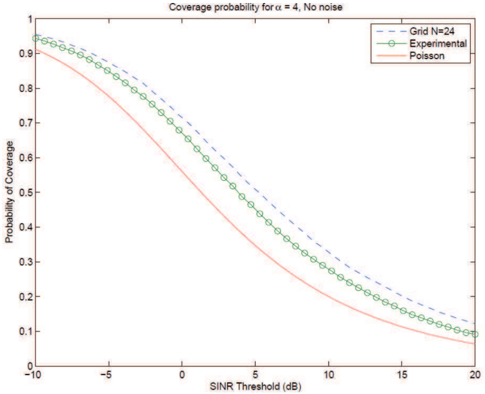
Actual 4G Deployment



Poisson Point Process (PPP)

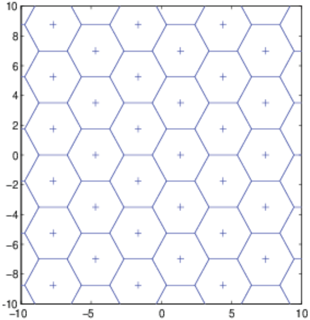
Stochastic Network Model is better model for 4G macrocells!

Modeling single-tier Cellular Network

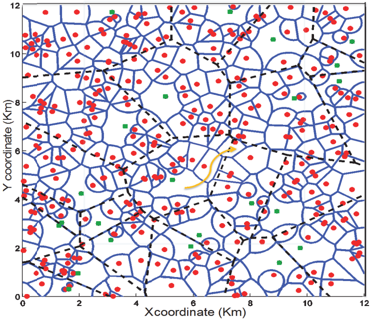


J. Andrews, F. Baccelli, and R. Ganti, “A tractable approach to coverage and rate in cellular networks”
IEEE Transactions on Communications, vol. 59, no. 11, 2011.

Modeling heterogenous Cellular Network

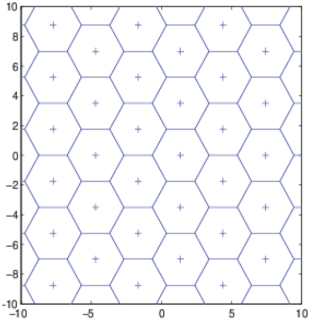


Grid Model

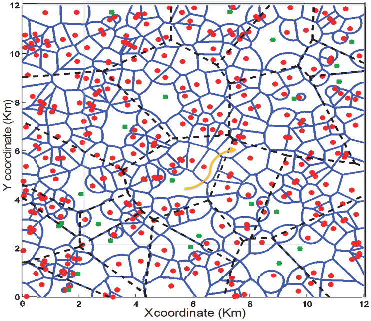


Two-tier Cellular network
(Two independent PPP)

Modeling heterogenous Cellular Network



Grid Model



Two-tier Cellular network
(Two independent PPP)

Stochastic Network Model is better model for heterogenous cellular networks.

The Need for Random Spatial Models

- To determine coverage, rate, reliability. Modeling HCNs?
- Fixed hexagonal model is fairly obsolete.
- Random spatial models are essential mathematical tools for modeling HCN (Stochastic Geometry).
- Wireless channels are modeled statistically (Rayleigh for small-scale variations and Lognormal for medium-scale).

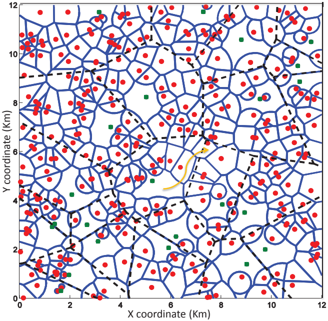


Figure: What is the coverage probability or average data rate?

Random Spatial Model

- K-tier network.
- Each tier has BS locations taken from independent Point Processes (PPP).
- BS densities: λ_k BS/m^2.
- Transmit power: P_k watts.
- SINR target: T_k (correct signal reception).
- Path loss exponent: α_k

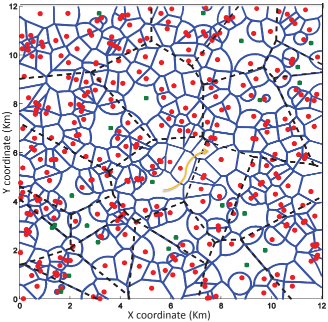
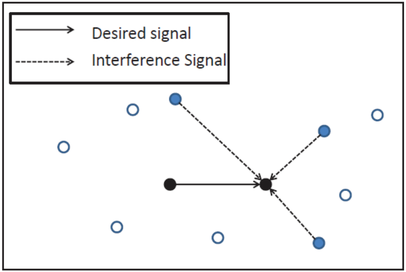


Figure: What is the coverage probability or average data rate?

SINR Model

$$SINR(y) = \frac{\overbrace{P_t(x_0) A h_{x_0 y} \| x_0 - y \|^{-\alpha}}^{\text{Desired signal}}}{\underbrace{W}_{\text{Noise}} + \underbrace{\sum_{x \in \mathcal{I}} P_t(x) A h_{xy} \| x - y \|^{-\alpha}}_{\text{Aggregation interference}}}$$



Statistic characteristics of aggregation interference signal power:

- Propagation model
- Topology (Interferers spatial distribution).
- Available channels.
- Traffic model.
- MAC layer protocol (Network operation model).
- Association model.

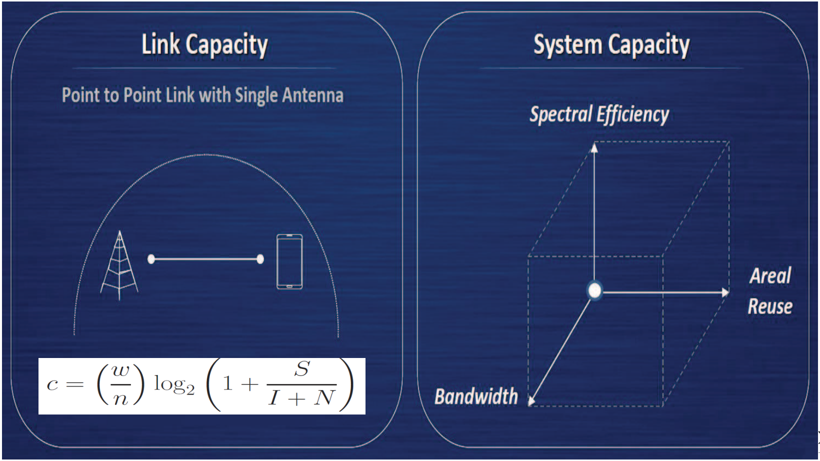
The Shannon Limit

The Shannon Limit is the maximum bit rate that a given communication channel can support.

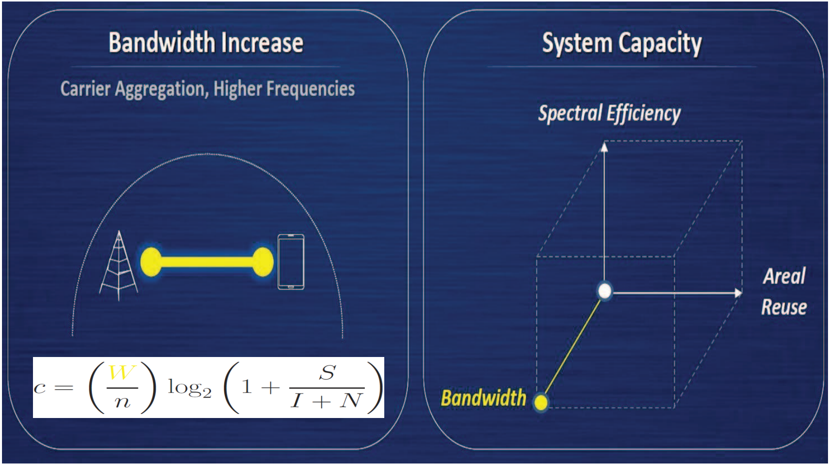
$$R < C = m\left(\frac{W}{n}\right) log_2\left(1+\frac{S}{I+N}\right) \qquad \text{[Bits/second]}$$

- *W* -- > BW (expensive).
- n -- > # of mobile users (BS load).
- M -->Spatial multiplexing factor (# of special streams).
- S -- > Desired signal power.
- I -- > Interference signal power.
- N -- > Noise signal power.

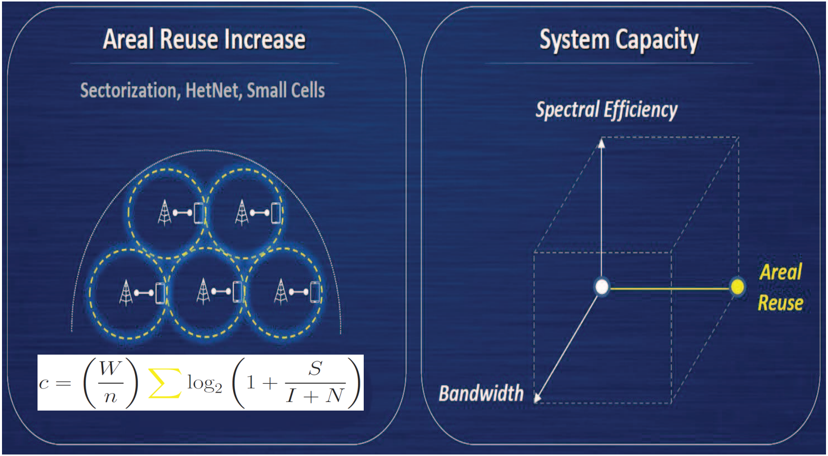
5G Capacity


Link Capacity
Point to Point Link with Single Antenna
$c = \left(\frac{w}{n}\right)\log_2\left(1+\frac{S}{I+N}\right)$
System Capacity
Spectral Efficiency
Areal Reuse
Bandwidth

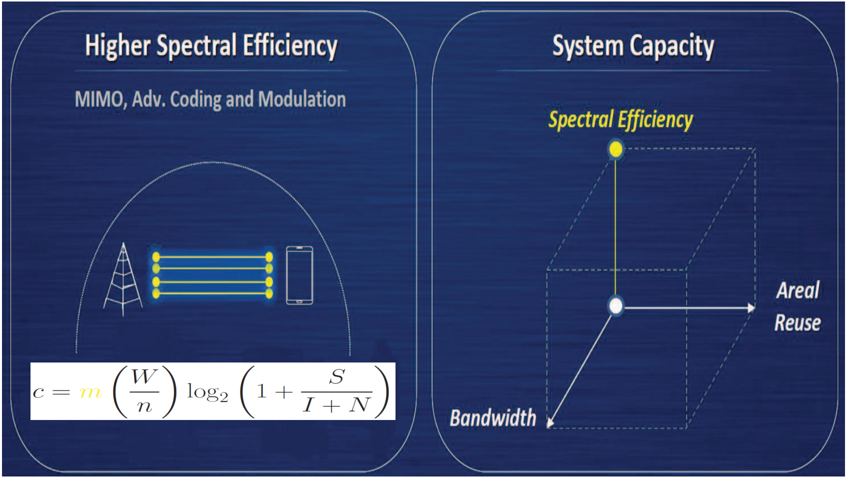
5G Capacity


Bandwidth Increase
Carrier Aggregation, Higher Frequencies
$c = \left(\frac{W}{n}\right) \log_2 \left(1 + \frac{S}{I + N}\right)$
System Capacity
Spectral Efficiency
Areal Reuse
Bandwidth

5G Capacity


Areal Reuse Increase
Sectorization, HetNet, Small Cells
$c = \left(\frac{W}{n}\right) \sum \log_2 \left(1 + \frac{S}{I+N}\right)$
System Capacity
Spectral Efficiency
Areal Reuse
Bandwidth

5G Capacity



$$c = m\left(\frac{W}{n}\right)\log_2\left(1+\frac{S}{I+N}\right)$$

Poisson point process

$\psi = \{x_i; i = 1, 2, 3, 4, \ldots\ldots\} \subset \mathbb{R}^d$ is a PPP with intensity λ if:

- # of points N(B) falling inside any compact set $B \subset \mathbb{R}^d$ is a Poisson random variable with a mean $\lambda \|B\|$.
- # of points in a disjoint sets are independent.

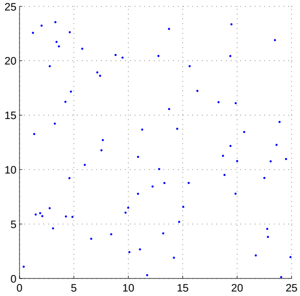


Figure: PPP in a 25m x 25m region with intensity 0.15 points/m2.

Poisson cluster process

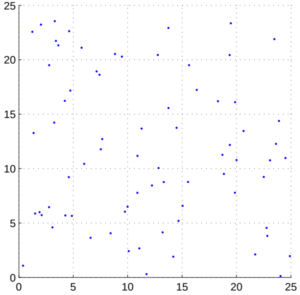


Figure: (a)PPP in a 25m x 25m region with intensity 0.15 points/m2.

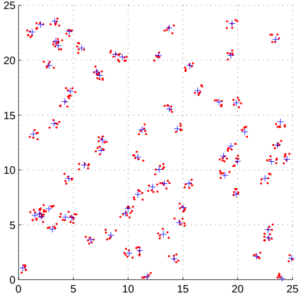


Figure: (b)PCP in a 25m x 25m region for the parent PPP in (a).

Hard core point process

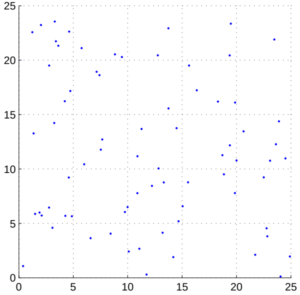


Figure: (a)PPP in a 25m x 25m region with intensity 0.15 points/m2.

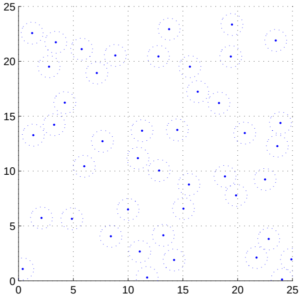


Figure: (b)Matern HCPP type I in a 25m x 25m region and hard core distance $\delta = 2$m.

Aggregation interference modeling

- Locations of interferers BSs are modeled as PP.
- Aggregation interference can be modeled as LT of *pdf* or *cdf*.
- No predefined expression for the *pdf* of the aggregation interference.
- Methods to model the aggregation interference:
    1. LT of the *pdf* of the aggregation interference.
    2. CF of the *pdf* of the aggregation interference.
    3. MGF of *pdf* of the aggregation interference.
- Stochastic geometry provides a predefined way to obtain the LT, or CF or the MGF of the aggregation interference which is associated with a specific point process.

Aggregation interference assumptions

How to tackle the problem of nonexistence of any useful closed-form expression for the pdf of the interference?

- BSs are distributed as a PPP and both desired and interfering signals are modeled as Rayleigh fading with channel power gain $h_{xy} \sim \exp(\mu)$.
- Approximate the *pdf* of the aggregation interference by one of the known *pdf* distributions. Then, LT, CF, and MGF can be used.
- Considering only the effect of nearest interferers assumption [dominant interferers and assuming a high pass loss exponent, i.e, $\alpha = 4$].

Coverage probability

$$
\begin{aligned}
P_c &= \mathbb{P}[\mathrm{SINR} > T] \\
&= \mathbb{P}\left[\frac{P h_{xy} r^{-\alpha}}{I_{agg} + \sigma^2} > T\right] \\
&= \mathbb{P}\left[h_{xy} > \frac{T(I_{agg} + \sigma^2) r^{\alpha}}{P}\right] \\
&= \int_v F_{h_{xy}}\left(\frac{(\sigma^2 + v) T r^{\alpha}}{P}\right) f_{I_{agg}} \, \mathrm{d}v \\
&\overset{(i)}{=} \mathbb{E}_{I_{agg}}\left[\exp\left(-\frac{(\sigma^2 + I_{agg})\, \mu T r^{\alpha}}{P}\right)\right] \\
&= \int_0^{\infty} \exp\left(\frac{-\sigma^2 T \mu r^{\alpha}}{P}\right) \mathbb{E}_{I_{agg}}\left[\exp\left(\frac{-I_{agg} T \mu r^{\alpha}}{P}\right)\right] f_R(r) \mathrm{d}r \\
&= \int_0^{\infty} \exp\left(\frac{-\sigma^2 T \mu r^{\alpha}}{P}\right) \mathscr{L}_{I_{agg}}(s)\Big|_{s = \frac{T\mu r^{\alpha}}{P}} f_R(r) \mathrm{d}r \\
&= \int_0^{\infty} \exp\left(\frac{-\sigma^2 T \mu r^{\alpha_1}}{P}\right) \mathscr{L}_{I_{agg}}\left(\frac{T \mu r^{\alpha}}{P}\right) f_R(r) \mathrm{d}r \\
&= \int_0^{\infty} \exp\left(\frac{-\sigma^2 T \mu r^{\alpha}}{P}\right) \mathscr{L}_{I_{agg}}(s) f_R(r) \mathrm{d}r \\
&\overset{(ii)}{=} \int_0^{\infty} \exp\left(-\sigma^2 c T\right) \mathscr{L}_{I_{agg}}(s) f_R(r) \mathrm{d}r
\end{aligned} \tag{1}
$$

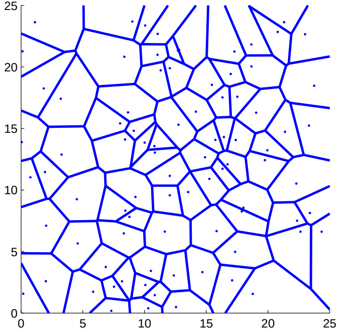
- BSs modeled as PPP.
- Desired and interfering signals are modeled as Rayleigh fading.
- Channel power gain $h_{xy} \sim \exp(\mu)$.
- Expectation value in (i) is with respect PP and channel gains.

Average data rate

$$
\begin{aligned}
\mathbb{E}[\ln(1+\mathrm{SINR})] &\overset{(i)}{=} \int_0^\infty \mathbb{P}[\ln(1+\mathrm{SINR}) > \zeta]\mathrm{d}\zeta \\
&= \int_0^\infty \mathbb{P}[\mathrm{SINR} > (e^{\zeta}-1)]\mathrm{d}\zeta \\
&\overset{(ii)}{=} \int_0^\infty \int_0^\infty \exp\left(-\sigma^2 c\left(e^{\zeta}-1\right)\right)\mathscr{L}_{I_{agg}}\left(\frac{\left(e^{\zeta}-1\right)\mu r^{\alpha}}{P}\right) f_R(r)\mathrm{d}r\mathrm{d}\zeta, \\
&= \int_0^\infty \int_0^\infty \exp\left(-\sigma^2 c\left(e^{\zeta}-1\right)\right)\mathscr{L}_{I_{agg}}(s)\Big|_{s=\frac{\left(e^{\zeta}-1\right)\mu r^{\alpha}}{P}} f_R(r)\mathrm{d}r\mathrm{d}\zeta, \\
&= \int_0^\infty \int_0^\infty \exp\left(-\sigma^2 c\left(e^{\zeta}-1\right)\right)\mathscr{L}_{I_{agg}}(s) f_R(r)\mathrm{d}r\mathrm{d}\zeta, \qquad (2)
\end{aligned}
$$

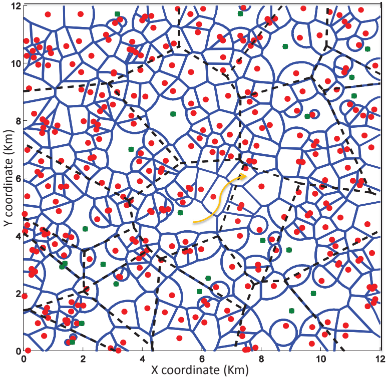
Single-tier cellular networks

- Coverage of BSs forms a Poisson Voronoi tessellation.
- P_c and data rate do not depend on BSs density [interference limited $(\sigma^2 = 0)$] [**?**].
- BSs intensity $\uparrow \Longrightarrow$ neither increase the coverage probability nor degrade it.
- Coverage probability $\uparrow \Longrightarrow$ Interference management (inter-cell cooperation and frequency reuse).



Multi-tier heterogenous cellular networks

- Small cells (micro BS, pico BSs, femto BSs, relays and DAS) $\Longrightarrow$ underlaid macro cellular.
- Macro BSs $\Longrightarrow$ 40 W.
- Small cells $\Longrightarrow$ 250 mW to 2 W.
- Mobile user association $\Longrightarrow$ maximum received power.
- Multi-tier cellular network $\Longrightarrow$ weighted Possion Voronoi tessellation.



Resource allocation and interference management

- Interference between neighboring small cells, and between small cells and a macrocell.
- Co-tier interference: between same layer network elements, i.e. inter-small cell interference or inter-macrocell interference.
- Cross-tier interference: between network elements that belong to the different tiers of the network, i.e. between small cells and macrocell.
- Distributed interference management scheme is required which satisfies the QoS requirements of the macrocell and small cell users and at the same time enhances the capacity and coverage of the network.

Resource allocation and interference management OFDMA-based BSs

- Small cells and macrocell $\Longrightarrow$ OFDMA-based BSs.
- BSs schedule resource blocks to mobile users using FDD and TDD.
- LTE and LTE-A $\Longrightarrow$ frequency reuse factor=1 ($\Uparrow$ spectrum efficiency).
- High inter-cell interfere for cell edge users.
- Interference coordination is mandatory to enhances the capacity and coverage of the network.
- Coordination between BSs $\Longrightarrow$ X2 interface.

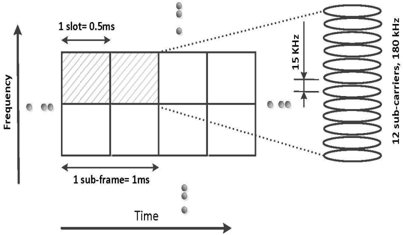
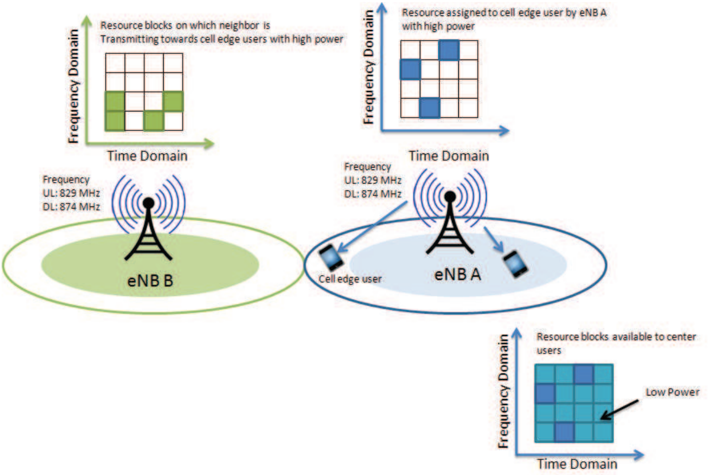
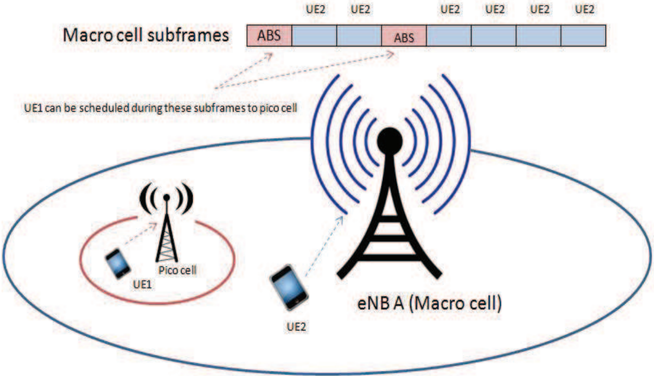


Figure: Resource blocks scheduling LTE downlink.

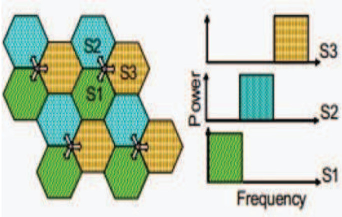
Intercell interference coordination (ICIC)



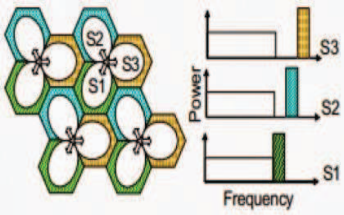
Enhanced intercell interference coordination (eICIC)



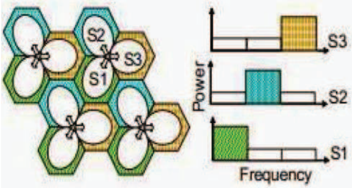
Frequency Reuse



A) Hard Frequency Reuse



B) Fractional Frequency Reuse

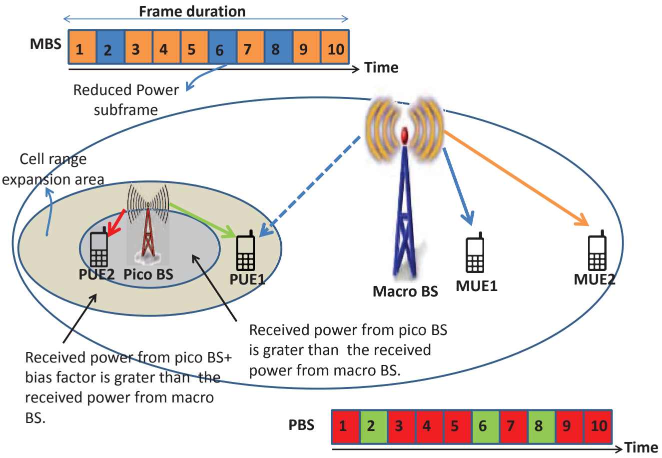


C) Soft Frequency Reuse

Spectrum Allocation

- The available spectrum bandwidth is divided into orthogonal groups of channels.
- The first group of channels is assigned to macro BSs and the second group of channels is assigned to small cells BSs.
- Allocation scheme (fixed/dynamic)? maximizes area spectral efficiency which guaranties a certain QoS for both macro mobile users and small cells mobile users.
- Spectrum allocation has to consider QoS, BSs densities in each tier, and spectral efficiency for mobile users in each tier.

Biasing and Load Balancing



Research Directions

- CP/UP split cellular network architecture (two-tier).

- Handover impact on the control signaling for CP/UP split cellular network architecture.

- Intelligent mobile users velocity aware offloading mechanism in two-tier C-plane/U-plane split RAN downlink which offloads mobile UEs from small cells to macro BS, while static UEs should be handed over to the ultra-dense small cell tier.

- Idle mode capability of small cells (turning off low load small cells) in two-tier C-plane/U-plane split RAN downlink 5G cellular networks to mitigate inter-cell interference and save energy.

Research Directions

- Handover failure probability under fluctuating channel conditions using handover system level parameters optimization, i.e, adaptive time to trigger and adaptive A3-offset.

- Study the effect WiFi and LTE coexistence in unlicensed spectrum 5 GHz (LTE-U).

Conclusion

- The future network architecture is heterogeneous with macro cells, small cells, along with WiFi.
- Random spatial models are essential mathematical tools for modeling modern cellular paradigm.
- Small cells architectures are going to be ubiquitous very soon.
- Small cells is one of the essential technologies to meet the 5G capacity requirements.

Questions

Questions?