Cellular Networks Modeling, Analysis, and Design Using Stochastic Geometry

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Outline

1. Motivation
2. Key Performance Indicators (KPIs)
3. Stochastic Point Processes
4. Modeling, Analysis, and Design of Cellular Networks
5. High Density Small Cells Heterogenous Cellular Networks
6. Research Directions and Conclusion
Motivation
Key Performance Indicators (KPIs)
Stochastic Point Processes
Modeling, Analysis, and Design of Cellular Networks
High Density Small Cells Heterogenous Cellular Networks
Research Directions and Conclusion

Background
Coverage Probability
Solution

Roadmap to 5G

1G
AMPS, TACS
1980s

2G
TDMA

3G
WCDMA, TD-SCDMA, CDMA2000
2000s

4G
LTE/LTE-A, 802.16m
2010s

OFDMA

SDN, CP/UP Split, Network Virtualization, Ultra Dense Small Cells, MIMO, C-RAN ……..

2020: 5G ??

Source: Cisco
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Source: Cisco

Heterogeneous cellular networks (micro BSs, pico BSs, femto BSs, and DAS underlaid a macro cellular network,)

SDN, CP/UP Split, Network Virtualization, Ultra Dense Small Cells, MIMO, C-RAN …….
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### Motivation

- Key Performance Indicators (KPIs)
- Stochastic Point Processes
- Modeling, Analysis, and Design of Cellular Networks
- High Density Small Cells Heterogenous Cellular Networks
- Research Directions and Conclusion

### Background

- Coverage Probability

### Solution

- Roadmap to 5G
  - **1G**
    - AMPS, TACS 1980s
  - **2G**
    - GSM, cdmaOne(IS-95) 1990s
  - **3G**
    - WCDMA, TD-SCDMA, CDMA2000 2000s
  - **4G**
    - LTE/LTE-A, 802.16m 2010s
  - **5G**
    - SDN, CP/UP Split, Network Virtualization, Ultra Dense Small Cells, MIMO, C-RAN

**Hexagonal Grid Model**

- **Stochastic Geometry Model**

Source: Cisco

Heterogeneous cellular networks (micro BSs, pico BSs, femto BSs, and DAS underlaid a macro cellular network.)
Modeling single-tier Cellular Network

Grid Model

Actual 4G Deployment

Poisson Point Process (PPP)
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Actual 4G Deployment
Poisson Point Process (PPP)

Stochastic Network Model is better model for 4G macrocells!
Modeling single-tier Cellular Network

Modeling heterogenous Cellular Network

Grid Model

Two-tier Cellular network
(Two independent PPP)
Stochastic Network Model is better model for heterogenous cellular networks.
The Need for Random Spatial Models

- To determine coverage, rate, reliability. Modeling HCNs?
- Fixed hexagonal model is fairly obsolete.
- Random spatial models are essential mathematical tools for modeling HCN (Stochastic Geometry).
- Wireless channels are modeled statistically (Rayleigh for small-scale variations and Lognormal for medium-scale).

Figure: What is the coverage probability or average data rate?
Random Spatial Model

- K-tier network.
- Each tier has BS locations taken from independent Point Processes (PPP).
- BS densities: $\lambda_k$ BS/m².
- Transmit power: $P_k$ watts.
- SINR target: $T_k$ (correct signal reception).
- Path loss exponent: $\alpha_k$

**Figure:** What is the coverage probability or average data rate?
SINR Model

\[
SINR(y) = \frac{P_t(x_0)Ah_{x_0y} \| x_0 - y \|^{-\alpha}}{\sqrt{W} + \sum_{x \in \mathcal{L}} P_t(x)Ah_{xy} \| x - y \|^{-\alpha}}
\]

# Statistic characteristics of aggregation interference signal power:
- Propagation model
- Topology (Interferers spatial distribution).
- Available channels.
- Traffic model.
- MAC layer protocol (Network operation model).
- Association model.
The Shannon Limit is the maximum bit rate that a given communication channel can support.

$$R < C = m \left( \frac{W}{n} \right) \log_2 \left( 1 + \frac{S}{I+N} \right) \quad \text{[Bits/second]}$$

- $W$ -- $\rightarrow$ BW (expensive).
- $n$ -- $\rightarrow$ # of mobile users (BS load).
- $M$ --$\rightarrow$ Spatial multiplexing factor (# of special streams).
- $S$ -- $\rightarrow$ Desired signal power.
- $I$ -- $\rightarrow$ Interference signal power.
- $N$ -- $\rightarrow$ Noise signal power.
5G Capacity

Link Capacity

Point to Point Link with Single Antenna

System Capacity

Spectral Efficiency

Areal Reuse

Bandwidth

\[ c = \left( \frac{w}{n} \right) \log_2 \left( 1 + \frac{S}{I + N} \right) \]
5G Capacity

Bandwidth Increase
Carrier Aggregation, Higher Frequencies

System Capacity
Spectral Efficiency
Areal Reuse
Bandwidth

\[ c = \left( \frac{W}{n} \right) \log_2 \left( 1 + \frac{S}{I + N} \right) \]
5G Capacity

Areal Reuse Increase
Sectorization, HetNet, Small Cells

\[ c = \left( \frac{W}{n} \right) \sum \log_2 \left( 1 + \frac{S}{I+N} \right) \]
5G Capacity

Higher Spectral Efficiency

MIMO, Adv. Coding and Modulation

System Capacity

Spectral Efficiency

Areal Reuse

Bandwidth

\[ c = m \left( \frac{W}{n} \right) \log_2 \left( 1 + \frac{S}{I + N} \right) \]
Poisson point process

\[ \psi = \{ x_i; i = 1, 2, 3, 4, \ldots \} \subset \mathbb{R}^d \] is a PPP with intensity \( \lambda \) if:

- \# of points \( N(B) \) falling inside any compact set \( B \subset \mathbb{R}^d \) is a Poisson random variable with a mean \( \lambda \|B\| \).
- \# of points in a disjoint sets are independent.

**Figure:** PPP in a 25m x 25m region with intensity 0.15 points/m².
**Poisson cluster process**

**Figure:** (a) PPP in a 25m x 25m region with intensity 0.15 points/m².

**Figure:** (b) PCP in a 25m x 25m region for the parent PPP in (a).
Motivation

Key Performance Indicators (KPIs)

Stochastic Point Processes

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Hard core point process

**Figure:** (a) PPP in a 25m x 25m region with intensity 0.15 points/m².

**Figure:** (b) Matern HCPP type I in a 25m x 25m region and hard core distance $\delta = 2m$. 
Aggregation interference modeling

- Locations of interferers BSs are modeled as PP.
- Aggregation interference can be modeled as LT of $pdf$ or $cdf$.
- No predefined expression for the $pdf$ of the aggregation interference.
- Methods to model the aggregation interference:
  1. LT of the $pdf$ of the aggregation interference.
  2. CF of the $pdf$ of the aggregation interference.
  3. MGF of $pdf$ of the aggregation interference.
- Stochastic geometry provides a predefined way to obtain the LT, or CF or the MGF of the aggregation interference which is associated with a specific point process.
Aggregation interference assumptions

How to tackle the problem of nonexistence of any useful closed-form expression for the pdf of the interference?

- BSs are distributed as a PPP and both desired and interfering signals are modeled as Rayleigh fading with channel power gain $h_{xy} \sim \exp(\mu)$.

- Approximate the pdf of the aggregation interference by one of the known pdf distributions. Then, LT, CF, and MGF can be used.

- Considering only the effect of nearest interferers assumption [dominant interferers and assuming a high pass loss exponent, i.e, $\alpha = 4$].
Coverage probability

\[ P_c = \mathbb{P}[\text{SINR} > T] \]

\[ = \mathbb{P} \left[ \frac{P h_{xy} r^{-\alpha}}{l_{agg} + \sigma^2} > T \right] \]

\[ = \mathbb{P} \left[ h_{xy} > \frac{T (l_{agg} + \sigma^2) r^\alpha}{P} \right] \]

\[ = \int_{v} F_{h_{xy}} \left( \frac{(\sigma^2 + v) T r^\alpha}{P} \right) f_{l_{agg}} dv \]

\[ (i) \mathbb{E}_{l_{agg}} \left[ \exp \left( -\frac{(\sigma^2 + l_{agg}) \mu T r^\alpha}{P} \right) \right] \]

\[ = \int_{0}^{\infty} \exp \left( -\frac{\sigma^2 T \mu r^\alpha}{P} \right) \mathbb{E}_{l_{agg}} \left[ \exp \left( -\frac{l_{agg} T \mu r^\alpha}{P} \right) \right] f_R(r) dr \]

\[ = \int_{0}^{\infty} \exp \left( -\frac{\sigma^2 T \mu r^\alpha}{P} \right) \mathcal{L}_{l_{agg}} (s) \bigg|_{s = \frac{T \mu r^\alpha}{P}} f_R(r) dr \]

\[ = \int_{0}^{\infty} \exp \left( -\frac{\sigma^2 T \mu r^\alpha_1}{P} \right) \mathcal{L}_{l_{agg}} \left( \frac{T \mu r^\alpha}{P} \right) f_R(r) dr \]

\[ = \int_{0}^{\infty} \exp \left( -\frac{\sigma^2 T \mu r^\alpha}{P} \right) \mathcal{L}_{l_{agg}} (s) f_R(r) dr \]

\[ (ii) \int_{0}^{\infty} \exp \left( -\sigma^2 c T \right) \mathcal{L}_{l_{agg}} (s) f_R(r) dr \]

- BSs modeled as PPP.
- Desired and interfering signals are modeled as Rayleigh fading.
- Channel power gain \( h_{xy} \sim \exp(\mu) \).
- Expectation value in (i) is with respect PP and channel gains.
Average data rate

\[
\mathbb{E}[\ln(1 + \text{SINR})] \overset{(i)}{=} \int_{0}^{\infty} \mathbb{P}[\ln(1 + \text{SINR}) > \zeta] d\zeta \\
= \int_{0}^{\infty} \mathbb{P}[\text{SINR} > (e^\zeta - 1)] d\zeta \\
\overset{(ii)}{=} \int_{0}^{\infty} \int_{0}^{\infty} \exp\left(-\sigma^2 c \left(e^\zeta - 1\right)\right) \mathcal{L}_{\text{agg}} \left(\frac{\left(e^\zeta - 1\right) \mu r^\alpha}{P}\right) f_R(r) dr d\zeta, \\
= \int_{0}^{\infty} \int_{0}^{\infty} \exp\left(-\sigma^2 c \left(e^\zeta - 1\right)\right) \mathcal{L}_{\text{agg}}(s) \left|_{s = \frac{(e^\zeta - 1) \mu r^\alpha}{P}}\right. f_R(r) dr d\zeta, \\
= \int_{0}^{\infty} \int_{0}^{\infty} \exp\left(-\sigma^2 c \left(e^\zeta - 1\right)\right) \mathcal{L}_{\text{agg}}(s) f_R(r) dr d\zeta, \tag{2}
\]
Coverage of BSs forms a Poisson Voronoi tessellation.

$P_c$ and data rate do not depend on BSs density [interference limited ($\sigma^2 = 0$)] [?].

BSs intensity $\uparrow \implies$ neither increase the coverage probability nor degrade it.

Coverage probability $\uparrow \implies$ Interference management (inter-cell cooperation and frequency reuse).
Multi-tier heterogenous cellular networks

- Small cells (micro BS, pico BSs, femto BSs, relays and DAS) \(\rightarrow\) underlaid macro cellular.
- Macro BSs \(\rightarrow\) 40 W.
- Small cells \(\rightarrow\) 250 mW to 2 W.
- Mobile user association \(\rightarrow\) maximum received power.
- Multi-tier cellular network \(\rightarrow\) weighted Possion Voronoi tessellation.
Interference between neighboring small cells, and between small cells and a macrocell.

Co-tier interference: between same layer network elements, i.e. inter-small cell interference or inter-macrocell interference.

Cross-tier interference: between network elements that belong to the different tiers of the network, i.e. between small cells and macrocell.

Distributed interference management scheme is required which satisfies the QoS requirements of the macrocell and small cell users and at the same time enhances the capacity and coverage of the network.
Resource allocation and interference management OFDMA-based BSs

- Small cells and macrocell $\implies$ OFDMA-based BSs.
- BSs schedule resource blocks to mobile users using FDD and TDD.
- LTE and LTE-A $\implies$ frequency reuse factor=1 ($\uparrow$ spectrum efficiency).
- High inter-cell interfere for cell edge users.
- Interference coordination is mandatory to enhances the capacity and coverage of the network.
- Coordination between BSs $\implies$ X2 interface.

**Figure**: Resource blocks scheduling LTE downlink.
Intercell interference coordination (ICIC)
Enhanced intercell interference coordination (eICIC)

UE1 can be scheduled during these subframes to pico cell
Frequency Reuse

A) Hard Frequency Reuse

B) Fractional Frequency Reuse

C) Soft Frequency Reuse
Spectrum Allocation

- The available spectrum bandwidth is divided into orthogonal groups of channels.
- The first group of channels is assigned to macro BSs and the second group of channels is assigned to small cells BSs.
- Allocation scheme (fixed/dynamic)? maximizes area spectral efficiency which guarantees a certain QoS for both macro mobile users and small cells mobile users.
- Spectrum allocation has to consider QoS, BSs densities in each tier, and spectral efficiency for mobile users in each tier.
Biasing and Load Balancing

Received power from pico BS is greater than the received power from macro BS. 

Frame duration

MBS

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Time

Reduced Power subframe

Cell range expansion area

Received power from pico BS + bias factor is greater than the received power from macro BS.

PBS

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Time
Research Directions

- CP/UP split cellular network architecture (two-tier).

- Handover impact on the control signaling for CP/UP split cellular network architecture.

- Intelligent mobile users velocity aware offloading mechanism in two-tier C-plane/U-plane split RAN downlink which offloads mobile UEs from small cells to macro BS, while static UEs should be handed over to the ultra-dense small cell tier.

- Idle mode capability of small cells (turning off low load small cells) in two-tier C-plane/U-plane split RAN downlink 5G cellular networks to mitigate inter-cell interference and save energy.
Research Directions

- Handover failure probability under fluctuating channel conditions using handover system level parameters optimization, i.e., adaptive time to trigger and adaptive A3-offset.

- Study the effect WiFi and LTE coexistence in unlicensed spectrum 5 GHz (LTE-U).
The future network architecture is heterogeneous with macro cells, small cells, along with WiFi.

Random spatial models are essential mathematical tools for modeling modern cellular paradigm.

Small cells architectures are going to be ubiquitous very soon.

Small cells is one of the essential technologies to meet the 5G capacity requirements.
Questions