EECS 3101 Fall 2017
Test 3 – Solutions to practice questions

1. The last question from the last tutorial.

   Solution: Done on the board.

2. 9.3-6 from the text.

   Hints: Divide and conquer – find the median of the k-th quantiles and partition with this as pivot. This takes linear time. Then recurse on each half to get the other quantiles.

3. 9.3-7 from the text.

   Hints: Find the median. Then compute the absolute difference of each element with the median. Find the k smallest numbers in this array using the linear time order statistic algorithm from Ch 9.

4. 9.3-9 from the text.

   Hints: Assume an odd number of stations. Show that if you shift the large pipeline up or down you increase the total cost of the vertical pipelines.

5. (a) Describe an algorithm that, given n integers in the range 0 to k, preprocesses its input and answers any query about how many of the n integers fall into a range $[1 \dots b]$ in $O(1)$ time. Your algorithm should use $\Theta(n+k)$ preprocessing time.

   Solution: This is done using a part of the counting sort algorithm. The cumulative counts in the $C[i]$ array in the counting sort algorithm indicates how many numbers are between 0 and i (inclusive) in the input. To get numbers in the range $1...b$ we can use $C[b] - C[0]$.

   (b) Show how to sort n integers in the range 0 to $n^4 - 1$ in $O(n)$ time.

   Solution: We express each integer in base n representation. Then the numbers are 4 digits long. We can do radix sort on these numbers in $O(n)$ time.

6. (a) The linear time algorithm to find the median of n numbers (this is the Selection algorithm in the text) uses groups of 5 elements in its computations. Why did it claim that at least $\frac{3n}{10} - 6$ elements were less than the median of medians it found?

   Solution: There were $\lceil n/5 \rceil$ groups created. Almost half of these groups have medians less than the median of medians – we leave out the last group (because it is possibly incomplete) and the group the median of medians came from. So we have $\lceil n/5 \rceil - 2$ such groups. Each of these groups contribute 3 elements that are less than the median of medians, or, at least $\frac{3n}{10} - 6 \leq$ elements were less than the median of medians.

   (b) Use the previous answer to explain each how each term on the right hand side of the recurrence : $T(n) \leq O(1)$ if $n < 140$ and

   $$T(n) \leq T(\lceil n/5 \rceil) + T(7n/10 + 6) + O(n), \text{ if } n \geq 140$$

   was obtained.

   Solution: See the book.

   (c) If you used groups of 7 instead of 5, what would the recurrence be?

   Solution:

7. Determine given strings X, Y, Z over the same alphabet if Z is a valid interleaving of X, Y.

   Solution:

   $T(n) \leq O(1)$ if $n < 126$ and

   $$T(n) \leq T(\lceil n/7 \rceil) + T(5n/7 + 8) + O(n), \text{ if } n \geq 126$$

8. Determine the shortest common superstring of given strings X, Y.

   Solution: We showed in the tutorial that the sum of the lengths of the shortest common superstring and the longest common subsequence equals the sum of the lengths of the two strings. We can use this fact to determine the length of the shortest common superstring. To get the shortest common superstring, we find the LCS and insert characters in the original strings between successive LCS characters in any order.