Next: Some mathematical tools

- Important to have the right tools
- Still, these are only tools; necessary but not sufficient to solve problems.
- We will cover some essential tools in this course for your repertoire.

A Quick Math Review

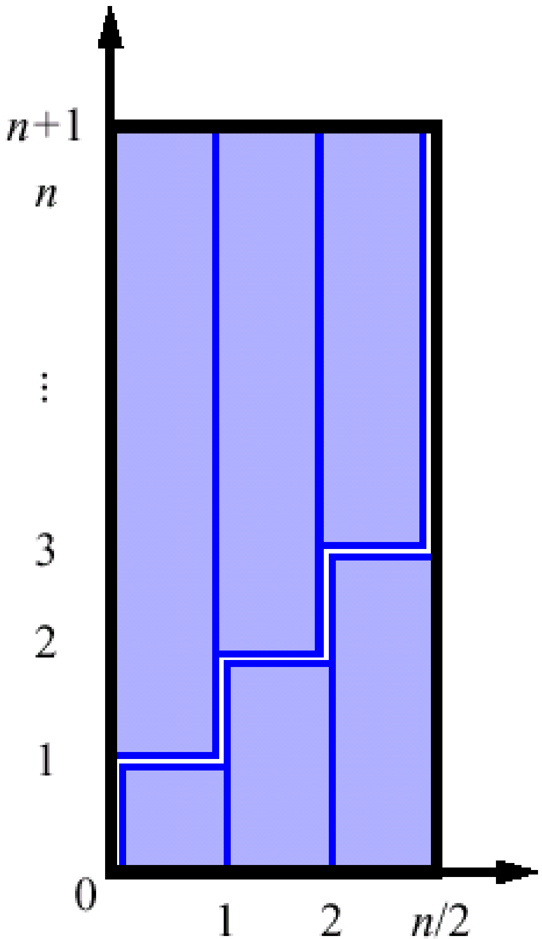
- Geometric progression
  - given an integer n_0 and a real number $0 < a \neq 1$

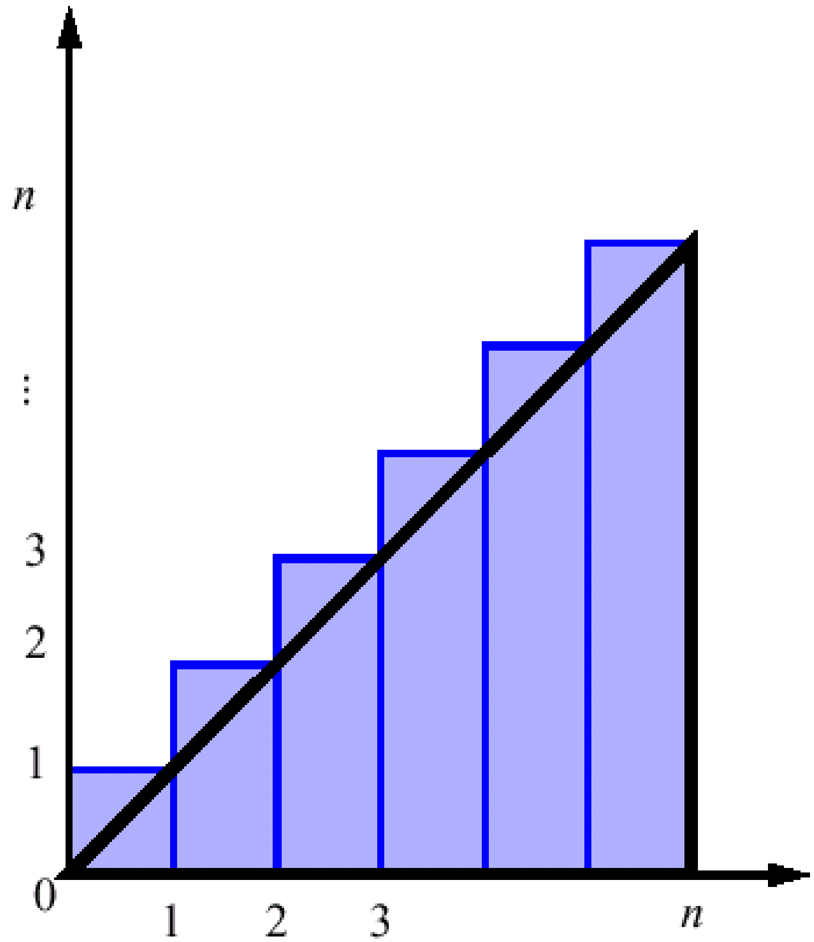
$$\sum_{i=0}^{n} a^i = 1 + a + a^2 + \ldots + a^n = \frac{1 - a^{n+1}}{1 - a}$$

  - geometric progressions exhibit exponential growth
- Arithmetic progression

$$\sum_{i=0}^{n} i = 1 + 2 + 3 + \ldots + n = \frac{n^2 + n}{2}$$

Pictorial proofs of sums





Review: Proof by Induction

- We want to show that property *P* is true for all integers $n \geq n_0$
- **Basis**: prove that *P* is true for n_0
- **Inductive step**: prove that if *P* is true for all *k* such that $n_0 \leq k \leq n-1$ then *P* is also true for *n*
- Example $S(n) = \sum_{i=0}^{n} i = \frac{n(n+1)}{2}$ for $n \geq 1$
- Base case: $S(1) = \sum_{i=0}^{1} i = \frac{1(1+1)}{2}$

Proof by Induction (2)

- Inductive Step

$$S(k)=\sum_{i=0}^{k} i=\frac{k(k+1)}{2} \text{ for } 1 \leq \mathrm{k} \leq n-1$$

$$S(n)=\sum_{i=0}^{n} i=\sum_{i=0}^{n-1} i+n=S(n-1)+n=$$

$$=(n-1)\frac{(n-1+1)}{2}+n=\frac{(n^2-n+2n)}{2}=$$

$$=\frac{n(n+1)}{2}$$

Important thumbrules for sums

"addition made easy" – Jeff Edmonds.

"Theta of last term"

Geometric like: $f(i) = 2^{\Omega(i)} \Rightarrow \sum_{i=1}^{n} f(i) = \Theta(f(n))$

no of terms x last term

Arithmetic like: $i.f(i) = i^{\Theta(1)} \Rightarrow \sum_{i=1}^{n} f(i) = \Theta(nf(n))$

Harmonic: $f(i) = 1/i \Rightarrow \sum_{i=1}^{n} f(i) = \Theta(\log n)$

"Theta of first term"

Bounded tail: $i.f(i) = 1/i^{\Theta(1)} \Rightarrow \sum_{i=1}^{n} f(i) = \Theta(1)$

Use as thumbrules only

Later: Some standard techniques

We will get into these techniques as and when we need them. If you are interested, read Appendix A.

- Approximation with integrals :Derive, rather than memorize the formula; e.g $\sum 1/k$.
- Telescoping sum: $\sum 1/(k(k+1))$
- Split a sum: $\sum k/2^k$
- Approximate crudely from both sides: e.g. $\sum 2^k$
- Integrate and differentiate series: $\sum kx^k$