EEECS4421Z: Introduction to Robotics

1. What is the forward kinematics problem for a robotic arm?

2. What is the inverse kinematics problem for a robotic arm?

3. Consider the arm shown in Figure 1.

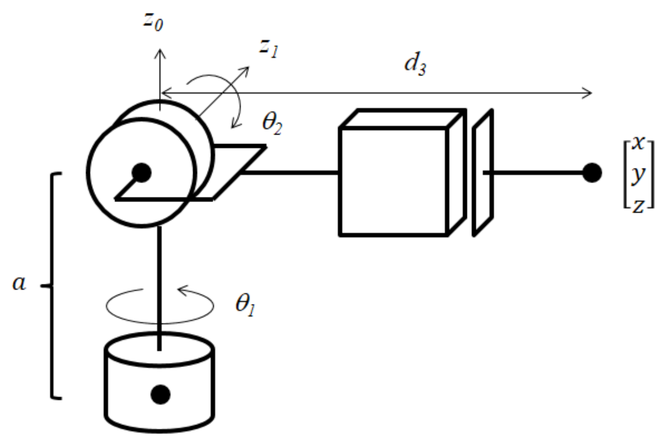


Figure 1: In this figure, θ_1 and θ_2 are shown at $0°$.

   (a) Solve the forward kinematics problem.

   (b) Given the location of the end effector $(x,\ y,\ z)^T$, solve the inverse kinematics problem (assume $d_3 \geq 0$).

4. Consider the arm shown in Figure 2.

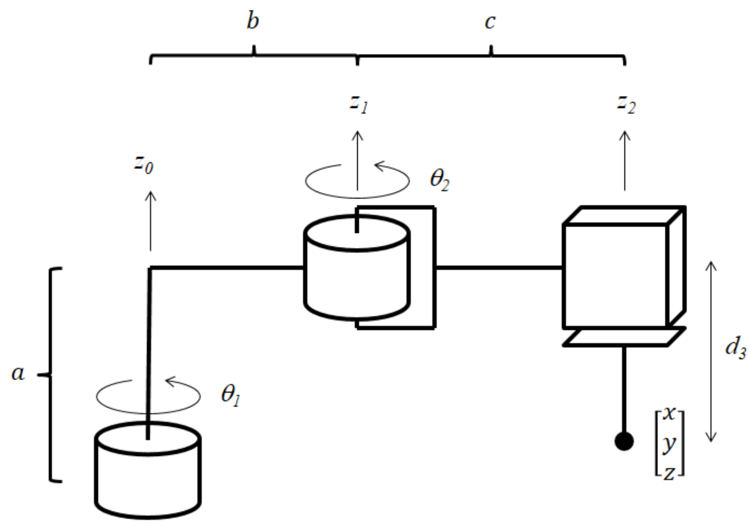


Figure 2: In this figure, θ_1 and θ_2 are shown at $0°$.

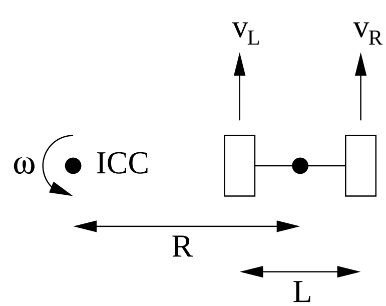
(a) Solve the forward kinematics problem.

(b) Given the location of the end effector $(x,\ y,\ z)^T$, solve the inverse kinematics problem (assume $d_3 \geq 0$).

5. Complete or add frames suitable for the Denavit-Hartenberg (DH) convention for the robot shown in Question 3, and give the table of DH parameters.

6. Complete or add frames suitable for the Denavit-Hartenberg (DH) convention for the robot shown in Question 4, and give the table of DH parameters.

7. Consider the differential drive mobile robot (shown below) with wheel spacing, L, left wheel ground velocity, v_L, right wheel ground velocity, v_R, angular velocity around the ICC, ω, and distance between the robot origin and the ICC, R.

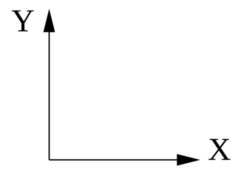


(a) Derive the equations for v_L and v_R in terms of ω, R, and L.

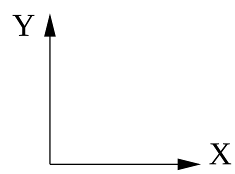
(b) A robotics student proposes a course project where they wish to modify an existing differential drive robot to make its left wheel steerable. What is your advice to the student? Justify your answer based on the kinematics of the differential drive.

8. (a) Sketch ellipses on the axes provided corresponding to the given covariance matrices. The exact dimensions of the ellipses are not important, but the sketches should be reasonable approximations of the true answers.

   i. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



   ii. $\begin{bmatrix} 3.25 & 1.299 \\ 1.299 & 1.75 \end{bmatrix}$



   Hint: the eigenvalues for the last matrix are 1 and 4.

   (b) Suppose you have two measurements $z_1 = 10$ and $z_2 = 20$ of the distance between a robot and an object. Assuming $z_1 \sim \mathcal{N}(0, \sigma_1^2)$ and $z_2 \sim \mathcal{N}(0, \sigma_2^2)$ what is the best estimate of the distance if:

   i. $\sigma_1 = \sigma_2$

   ii. $3\sigma_1 = \sigma_2$

9. What quantity does the velocity and odometry motion models compute?

10. Why is it more difficult to compute a motion model than it is to sample from a motion model?

11. Suppose that a robot is equipped with a sensor for measuring range and bearing to a landmark, and assume that the sensor can determine which landmark it is currently measuring. Assume that we are performing global localization with an extended Kalman filter. When measuring a single landmark, the probability distribution of the robot location is usually poorly approximated by a Gaussian distribution. When measuring two or more landmarks, the probability distribution of the robot location is usually well approximated by a Gaussian distribution. Explain why (hint: consider the case where the position and heading of the robot are highly uncertain).