

CSE4421Z: Introduction to Robotics

Final Exam

Instructor: Dr. Burton Ma

Thu 12 April 2012

Name: _____

Student Number: _____

Instructions

1. You have 2 hours to complete the exam.
2. Write your answers clearly and succinctly in the space provided on the question sheets; use the back of the page and the extra blank page if you need additional space for your answer.
3. Textbook, notes, and a calculator are permitted.
4. This exam has 1 cover page, 12 pages for questions and answers, and one copy of Section 5.4 of the textbook.

1. Consider the RR arm shown in Figure 1.

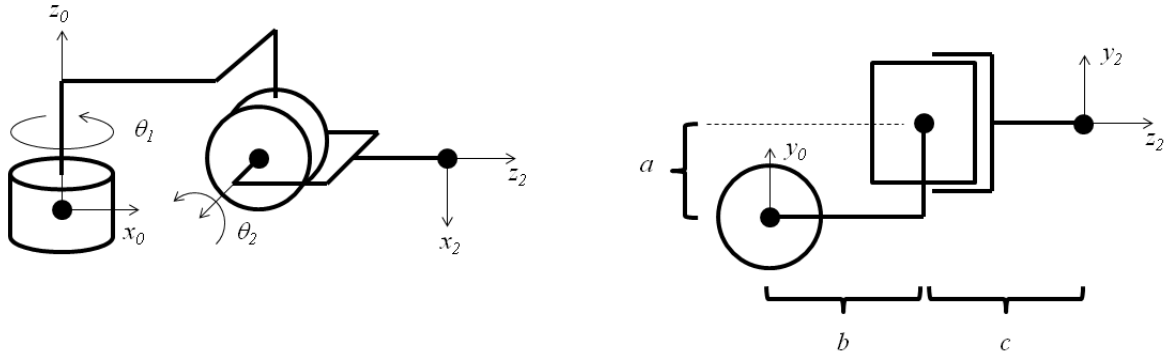


Figure 1: Left: Front view of arm. Right: Top-down view of arm. In this figure, all joint angles are shown at 0° .

Joints 1 and 2 are connected by a link with a 90° bend; the bend allows both joints to rotate through 360° without colliding with each other. The axis of joint 2 is always in the plane $z_0 = 0$. You may assume that the link dimensions a , b , and c are always greater than zero, and that $b > c$.

- (a) (22 points) Given the location of frame $\{2\}$ expressed in frame $\{0\}$, $o_2^0 = (x, y, z)^T$, find the values of θ_1 and θ_2 .
- (b) (3 points) A high quality solution to part (a) would check that the point $o_2^0 = (x, y, z)^T$ is actually reachable by the robot; how would you check if o_2^0 is reachable? Provide a mathematical expression if possible, although a well written description could also receive full marks.

Use this page for Question 1 if necessary.

2. (25 points) Consider the RR arm shown in Figure 2; this arm is similar to the arm from Question 1 except that the link has a bend of -45° .

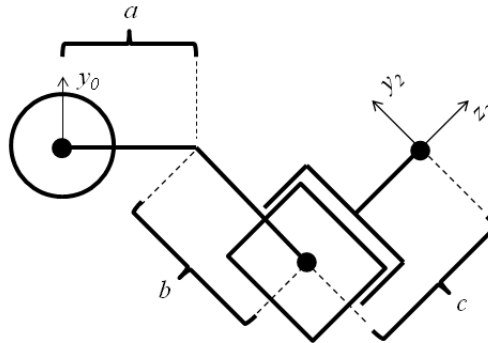


Figure 2: Top-down view of arm.

The axis of joint 2 is always in the plane $z_0 = 0$. You may assume that the link dimensions a , b , and c are always greater than zero.

Solve for the forward kinematics of the arm; that is, given joint angles θ_1 and θ_2 , find the orientation and position of frame $\{2\}$ as a 4×4 homogeneous matrix. If your solution involves a sequence of matrix multiplications then you do not need to perform the actual multiplications. Use of the Denavit-Hartenberg convention is acceptable, but not required.

Use this page for Question 2 if necessary.

3. For this question, consider the odometry motion model for a differential drive robot described in Section 5.4 of the textbook and Day 18 of the lecture slides. Also, consider the bicycle model from Assignment 3 (Exercise 5.8.4 of the textbook and slide 12 from Day 12 of the lecture slides) which has a steerable front wheel; assume that the bicycle has a motor that supplies power to the *front* wheel.
- (a) (5 points) The odometry motion model decomposes the motion into 3 steps: an in-place rotation, followed by a translation, followed by an in-place rotation. Is this a reasonable motion model for a bicycle? Briefly explain why or why not; one to three sentences should suffice. Consider reading part (b) before writing down your answer to this part.
- (b) (5 points) Assume that you are determined to use the odometry motion model from part (a); describe what the rider of the bicycle would have to do to duplicate the motion prescribed by the motion model (i.e., how would the rider have to turn the front wheel and how far would the bicycle have to travel for each of the three steps?).

- (c) (15 points) Propose an alternate odometry motion model where the control inputs are described by the control vector

$$u_t = \begin{bmatrix} \alpha_{t-1} \\ \alpha_t \\ d_t \end{bmatrix}$$

where α_{t-1} is the angle of the front wheel at time $t - 1$, α_t is the angle of the front wheel at time t , and d_t is the distance travelled by the front wheel; all of the control inputs are measured by sensors in the bicycle (e.g., rotary encoders in the steering wheel and front wheel). You can assume that the time steps between odometry measurements is small so that $|\alpha_t - \alpha_{t-1}|$ is small.

For your answer, you should first briefly describe your motion model (what is the assumed motion of the bicycle, and how is the motion decomposed into separate steps if necessary). Second, you should provide an algorithm to compute the probability density of the motion (similar to the algorithm given in Table 5.5 of the textbook).

Use this page for Question 3 if necessary.

4. Consider a differential drive robot moving in a planar environment, as shown in Figure 3.

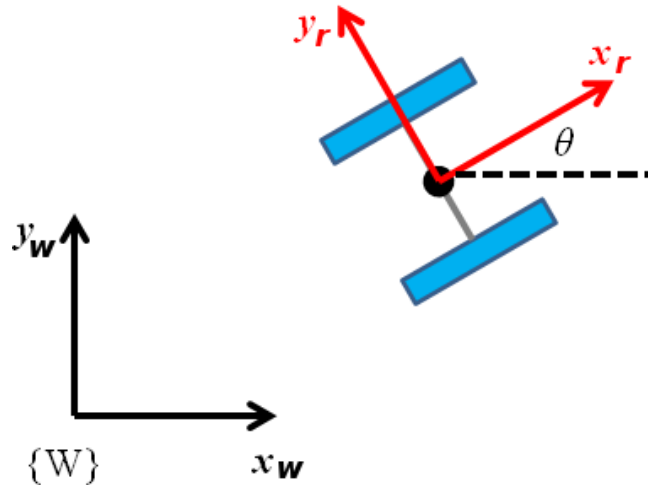


Figure 3: Differential drive moving in a planar environment with a world coordinate frame $\{W\}$. The position of the robot at time t in the world frame is $(x, y)^T$ and the heading of the robot at time t is θ . The robot has its own internal reference frame with the x axis of the frame pointing in the heading direction.

A map of the environment (in the world frame) is shown in Figure 4; there are three point landmarks in the environment.

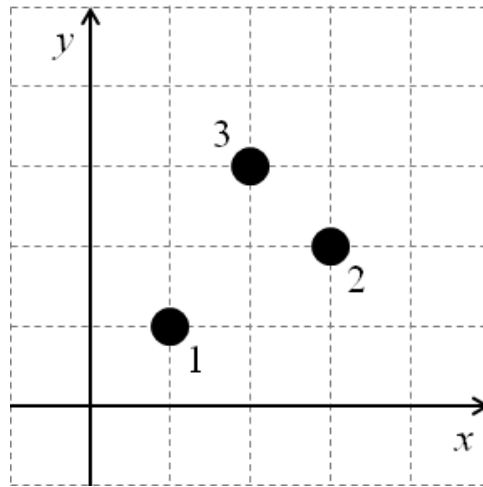
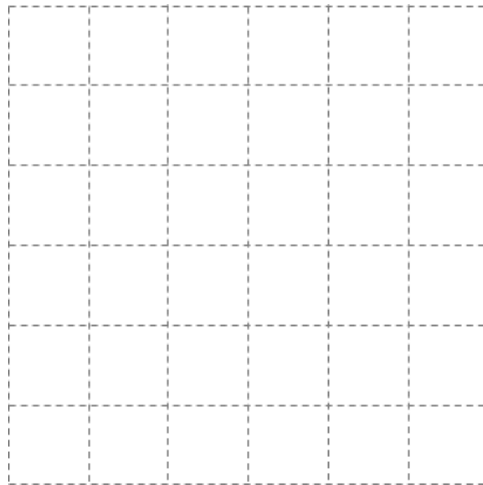


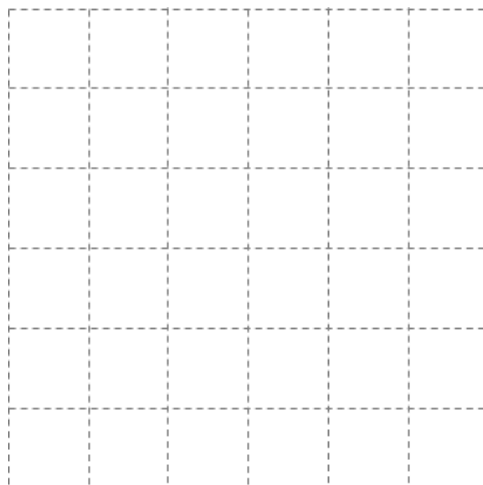
Figure 4: Map of the environment with three point landmarks.

Suppose that the robot can measure the location of all of the landmarks, with the coordinates of each landmark expressed in the internal reference frame of the robot (not the world frame).

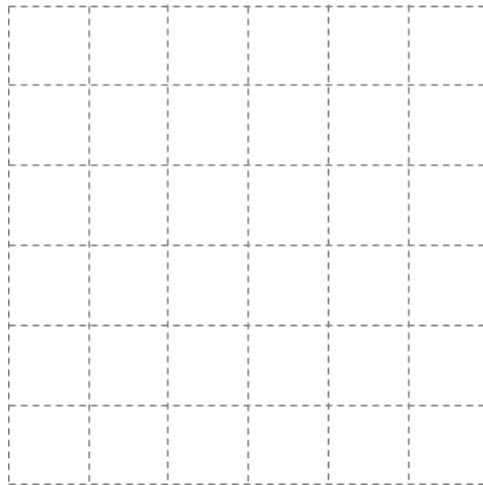
- (a) (5 points) Suppose that the robot pose at time t is $(-1, 2, 0^\circ)^T$. Draw the (noise free) landmark locations in the robot's internal reference frame; make sure to include the x and y axes of the robot's reference frame.



- (b) (5 points) Suppose that the robot pose at time t is $(0, 0, 45^\circ)^T$. Draw the (noise free) landmark locations in the robot's internal reference frame; make sure to include the x and y axes of the robot's reference frame.



- (c) (5 points) Suppose that the robot pose at time t is $(2, -1, 90^\circ)^T$. Draw the (noise free) landmark locations in the robot's internal reference frame; make sure to include the x and y axes of the robot's reference frame.



(d) (10 points) Let the landmark locations in the world be

$$\begin{aligned}M_1 &= (M_{1,x}, M_{1,y})^T \\M_2 &= (M_{2,x}, M_{2,y})^T \\M_3 &= (M_{3,x}, M_{3,y})^T\end{aligned}$$

Let the state of the robot at time t be

$$x_t = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

Let the measurement at time t be

$$z_t = \begin{bmatrix} m_{1,x} \\ m_{1,y} \\ m_{2,x} \\ m_{2,y} \\ m_{3,x} \\ m_{3,y} \end{bmatrix}$$

where $(m_{i,x}, m_{i,y})^T$ is the noisy measured location of landmark i in the robot's internal reference frame. Give a measurement model (suitable for use in an EKF or UKF) that relates the measurements z_t to the state of the robot x_t .

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