Homework Assignment #3
Due: October 18, 2016 at 1:00 p.m.

1. Consider an asynchronous ring of processes. Each process has a unique id, but does not initially know the ids of any other processes or even how many processes there are. There are no failures. The following two different versions of Leader Election were discussed in class.

LE: Every process must output 0 or 1. Exactly one process must output 1.

LE′: Every process must output the minimum id of any process in the system.

We showed in class that every algorithm for LE′ uses $\Omega(n \log n)$ messages in the worst case (where n is the number of processes in the ring). Explain why it follows from this that every algorithm for LE must use $\Omega(n \log n)$ messages in the worst case.

2. Consider a synchronous, anonymous model where processes are arranged in a ring. Each process receives an input bit (0 or 1). The goal is to compute the xor of all the bits.

(a) Suppose processes do not know the exact size of the ring, but they know that it is either n or $n + 1$. Prove that it is impossible to solve the problem, even if the system is synchronous.

(b) Now suppose processes know that the size of the ring is exactly n. Give a deterministic algorithm to compute the xor in an asynchronous system. How many messages does your algorithm use in the worst case? (The fewer, the better.)