# Review Questions on Mathematical Prerequisites

**1.** Prove that $\neg(\neg p \Leftrightarrow (r \vee p))$ is logically equivalent to $r \Rightarrow p$.

**2.** Use a proof by contradiction to show that if $n$ is an integer and $n^2$ is even, then $n$ is even.

**3.** Is the following statement true or false?
$\forall x$ in $\mathbb{R}, \exists y$ in $\mathbb{R}$ such that $y \geq 0 \wedge (y = x \vee y = -x)$.
Explain why your answer is correct.

**4.** Is the following statement true or false?
For all sets $A, B$ and $C$, $A - (B \cup C) \subseteq (A - B) \cap (A - C)$.
Prove your answer is correct.

**5.** Let $f : B \to C$ and $g : A \to B$. Prove that if $f \circ g$ is onto then $f$ is onto.

**6.** Prove that for every positive integer $n$, $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$.

**7.** Yark University has 40000 students. Each student takes 5 classes each term. The University offers 1000 classes each term. The largest classroom at Yark holds 180 students. Is this a problem? Explain why.

**8.** Consider the domain of all people.
Let $P(x, y)$ represent the statement "$x$ is the parent of $y$".

**(a)** Translate the following formulas into clear and precise English.

$\exists x \forall y P(x, y)$

$\forall y \exists x P(x, y)$

**(b)** Express the statement "Somebody has no grandchildren" using only the predicate $P$.

**9.** Prove that for all integers $a, b$ and $c$, the product of some pair of the three integers is non-negative. (In other words, show that $ab \geq 0$ or $ac \geq 0$ or $bc \geq 0$.)

**10.** Let $A = \{0, 1, 2\}$ and $B = \{1, 3\}$. List the elements of each of the following sets.

**(a)** $A - B =$

**(b)** $A \times B =$

**11.** Let $f : A \to B$. Let $S$ and $T$ be subsets of $A$.

**(a)** Prove that $f(S \cap T) \subseteq f(S) \cap f(T)$.

**(b)** Give an example of a function $f$ and sets $S$ and $T$ such that $f(S \cap T) \neq f(S) \cap f(T)$. Briefly explain why your answer is correct.

**12.** Prove $n^2 \leq 2^n$ for all natural numbers $n \geq 4$.

**13.** Let $f\colon A \to B$ be a function. For any set $C \subseteq B$, define $f^{-1}(C)$ to be the set $\{a \in A : f(a) \in C\}$. Prove that for every $f\colon A \to B$ and subsets $S$ and $T$ of $B$ we have $f^{-1}(S) \cap f^{-1}(T) = f^{-1}(S \cap T)$.

**14.** Show that if 51 distinct numbers are chosen among $\{1, 2, 3, \ldots, 100\}$ then there must be two numbers among them whose sum is 101.