Homework Assignment #9
Due: November 24, 2016 at 4:00 p.m.

1. If L is a language over the alphabet Σ, let $ROOT(L) = \{x \in \Sigma^* : \exists n \in \mathbb{N} \text{ such that } x^n \in L\}$.

(a) Let $L_a = \{x \in \{\mathtt{a}, \mathtt{b}\}^* : \text{ the number of } \mathtt{a}\text{'s in } x \text{ is a multiple of } 4\}$. Give a regular expression for $ROOT(L_a)$.

(b) Show that the set of recognizable languages is closed under the $ROOT$ operation. In other words, show that for every recognizable language L, $ROOT(L)$ is also recognizable.

(c) In this part, you will show that the set of decidable languages is not closed under $ROOT$. Recall that if M is a Turing machine, then $\langle M \rangle$ is a binary string that describes M. (This string is obtained by writing M in YUTMFF and then converting each character in the file to a string of 8 bits using ASCII codes.) Let

$$L_c = \{\langle M \rangle \# x : x \in \{\mathtt{0}, \mathtt{1}, \#\}^* \text{ and } M \text{ is a Turing machine that accepts } \varepsilon \text{ within } |x| \text{ steps}\}.$$

(i) Prove that L_c is decidable.

(ii) Let $EPS_{TM} = \{\langle M \rangle : M \text{ is a Turing machine that accepts } \varepsilon\}$. By Rice's Theorem, we know that EPS_{TM} is undecidable.

Prove that $ROOT(L_c)$ is undecidable. Hint: use a reduction involving EPS_{TM}.