# Homework Assignment #7
# Due: November 8, 2016 at 4:00 p.m.

**1.** In class, we discussed a way to simulate a $k$-tape Turing machine using an ordinary (single-tape) Turing machine. Our construction was a little different from the one given in the textbook's proof of Theorem 3.13. We divided the simulator's single tape into $k$ "tracks" so that each square of the single tape stored $k$ symbols of the simulated $k$-tape machine (one from each tape). To represent the location of the simulated $k$-tape TM on each tape, we circled the character at the simulated machine's current location.

In our construction, if the simulated machine had tape alphabet $\Gamma$ and input alphabet $\Sigma$, the tape alphabet of the simulator was $\Gamma' = (\Gamma \cup \textcircled{\Gamma})^k \cup \Sigma \cup \{\sqcup, \triangleright\}$, where $\textcircled{\Gamma} = \{\textcircled{x} : x \in \Gamma\}$.

**(a)** Suppose $\Sigma = \{\mathtt{0}, \mathtt{1}\}$ and $k = 3$. When the single-tape simulator starts, it has some input string $x$ in $\Sigma^*$ on the tape, followed by blanks. However, the initial state of the simulated 3-tape machine would have $x$ on tape 1 and blanks on tapes 2 and 3.

Draw a fragment of the simulator TM's transition diagram that performs the preprocessing required to transform the contents of the single-tape TM's tape into a representation of the initial state of the three tapes of the 3-tape machine.

For example, if $x = \mathtt{001}$, the simulator's tape initially contains

$$\triangleright\ \mathtt{0}\ \mathtt{0}\ \mathtt{1}\ \sqcup\ \sqcup\ \sqcup \ldots$$

After running this through the preprocessing stage, the simulator's tape would contain

$$\triangleright\ (\triangleright, \triangleright, \triangleright)\ (\textcircled{\mathtt{0}}, \textcircled{\mathtt{0}}, \textcircled{\mathtt{0}})\ (\mathtt{0}, \sqcup, \sqcup)\ (\mathtt{1}, \sqcup, \sqcup)\ \sqcup\ \sqcup\ \sqcup \ldots$$

(Your transition diagram should work for every string $x$, not just this example.)

**(b)** After the preprocessing stage, the single-tape TM begins to simulate the $k$-tape TM step by step. Describe, in English, what the single-tape simulation should do if it ever reaches a square that contains $\sqcup$.

**(c)** If the input string $x$ has length $n$, and the $k$-tape TM runs for $T$ steps, then the simulation on a single-tape TM will take $O(n + T^2)$ steps. (The $n$ steps are needed for the preprocessing stage and $O(T)$ steps are used to simulate each of the $T$ steps of the $k$-tape machine, as we discussed in class.)

Describe, in a couple of sentences, how the preprocessing stage could be avoided so that the entire simulation runs in $O(T^2)$ steps on the single-tape machine.

**2.** Let $\mathbb{N}$ be the set of all natural numbers.

**(a)** Prove that the set of all finite subsets of $\mathbb{N}$ is countable.

**(b)** Prove that the set of all subsets of $\mathbb{N}$ (i.e., $\mathscr{P}(\mathbb{N})$) is uncountable.