York University EECS 2001 September 30, 2016

Homework Assignment #4
Due: October 6, 2016 at 4:00 p.m.

1. If $L \subseteq \Sigma^*$ is any language, we define $ADD(L)$ to be the set of all strings that can be obtained by inserting a single character into a string in L. More formally,

$$ADD(L) = \{xay : xy \in L \text{ and } a \in \Sigma\}.$$

(a) Let $L_a = \{\texttt{01}, \varepsilon, \texttt{11}\}$. Here, $\Sigma = \{\texttt{0}, \texttt{1}\}$. List all the elements of $ADD(L_a)$.

(b) Let L_b be the language described by the regular expression $\texttt{a}(\texttt{bc})^*$. Write a regular expression for the language $ADD(L_b)$. Here, $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$. You do not have to prove your answer is correct.

(c) Your goal is to define a function D that maps regular expressions to regular expressions. Give a recursive definition of $A(R)$ so that if R is a regular expression for a language L, then $A(R)$ is a regular expression for the language $ADD(L)$.

You can write your answer by filling in the blanks in the following. (I have given you the answer for two of the six cases needed for the recursive definition.)

$$\begin{aligned} A(\emptyset) &= \emptyset \\ A(\varepsilon) &= \underline{\hspace{3cm}} \\ A(a) &= \underline{\hspace{3cm}}, \text{ for any } a \in \Sigma \end{aligned}$$

If R_1 and R_2 are regular expressions then,

$$\begin{aligned} A(R_1 \cup R_2) &= A(R_1) \cup A(R_2) \\ A(R_1 \cdot R_2) &= \underline{\hspace{3cm}} \\ A(R_1^*) &= \underline{\hspace{3cm}} \end{aligned}$$

(d) Prove that for all regular expressions R, if L is the language represented by R, then the language represented by $A(R)$ is $ADD(L)$.

In other words, explain why your answer to part (c) is correct for all R.

Hint: use mathematical induction on the number of operators $(\cup, ^*, \cdot)$ that appear in the regular expression R.

(e) Prove that if L is any regular language then $ADD(L)$ is also a regular language.