

Homework Assignment #3

Due: September 29, 2016 at 4:00 p.m.

Hand in Question 1 on paper in class. Submit Question 2 electronically.

1. Nine people are arranged in a circle, facing the centre, to play a game. At any time during the game, each person can either be standing up or sitting down. There is a clock on the wall. When the game starts at noon, at least one person is standing and at least one person is sitting. During the game, each person follows these instructions repeatedly:
 - Look to your left.
 - If you are sitting and the person to your left is standing, then stand up when the clock reaches the next minute.
 - If you are standing and the person to your left is standing, then sit down when the clock reaches the next minute.
 - Otherwise, do not change your position when the clock reaches the next minute.

The game ends when everyone is sitting down at the same time. (It is not a very fun game.)

For example, here is one possible way the game could begin, where 0 represents a person sitting down and 1 represents a person standing up. The people are listed in clockwise order around the circle.

```
12:00 101101000
12:01 110111001
12:02 011001010
12:03 101011110
12:04 111100011
12:05 000100100
12:06 001101100
12:07 010110100
      ⋮
```

- (a) Prove that, at every time, there must be either two adjacent standing people or two adjacent sitting people.
 - (b) Prove that, at every time, somebody is sitting.
 - (c) Prove that, no matter how the nine people are arranged at noon, the game will never end.
2. Consider the alphabet $\left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\}$. Let L be the language of all strings that represent correct steps of the game described in the previous question (for any number of players, not just nine). The top row of bits represents the people before the step, and the bottom row represents the people after the step (one minute later). For example, the string

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is in L because this string describes the step that took place in the example game above at 12:05. The string $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is also in the language, because if four people are standing up one minute, they will all be sitting down the next minute.

Note that there must be at least two people to play the game, so no string of length 1 is in L .

Construct a DFA that accepts L . Submit a YUFAFF file called `q2.txt` that describes your DFA. (YUFAFF was described in Assignment 2.) Use 0/0, 0/1, 1/0 and 1/1 to represent $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, respectively. Thus, the second line of your YUFAFF file should be:

0/0 0/1 1/0 1/1

Submit your YUFAFF file electronically in the same way that you submitted Assignment 2. If you are working with a partner, the two of you should only submit one YUFAFF file, but please also submit a file called `declaration.txt` that says who your partner is.