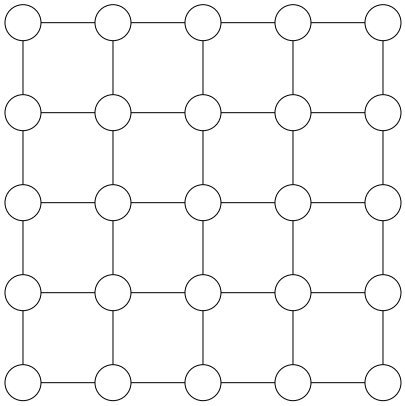
Homework Assignment #3
Due: February 3, 2016 at 5:30 p.m.

1. Consider a network of n^2 processes arranged in a grid pattern as shown below.



Use $P_{i,j}$ to denote the process in row i and column j (for $0 \leq i < n$ and $0 \leq j < n$). Channels are reliable and deliver messages in FIFO order. Processes are deterministic and anonymous (no id's, identically programmed). Processes know the value of n.

Initially, nodes have no sense of direction. This means that each node initially knows how many neighbours it has, but does not know which channel connects it to which neighbour. Thus if a node has four neighbours, it can send a message along channel #1, #2, #3 or #4, but it does not know whether channel #1 goes to its neighbour to the north, south, west or east. Any algorithm designed for this model must work correctly, regardless of how the local labels for the channels are assigned at each node.

In this problem we consider the version of leader election where exactly one process must output "leader" and all others must output "non-leader".

(a) Show that leader election is possible using 0 messages when $n = 3$.

(b) Give a careful proof showing that leader election is impossible when $n = 4$, even if the network is synchronous.

(c) Give necessary and sufficient conditions on n for leader election to be solvable in a synchronous system.

(i) Give a leader algorithm algorithm that solves leader election for values of n that satisfy your condition. How many messages does your algorithm use? (State your answer as a function of n using Θ notation.) The fewer messages, the better

(ii) Explain why your condition is necessary.

(d) Bonus part: Is there an *asynchronous* leader election algorithm for all values of n that satisfy your condition in part (c)?