

Homework Assignment #1

Due: January 13, 2016 at 5:30 p.m.

1. Recall the Two Generals problem from class. We shall consider a more general problem, where there are n generals instead of two. The generals have synchronized watches and each general begins with an initial preference for what time to attack. After communicating with one another according to some algorithm, the generals must decide what time to attack. The algorithm should satisfy the following properties.
 - Agreement: In every execution of the algorithm, the generals' decisions must always be identical.
 - Validity: If all the generals start with the same preference and there are no communication failures, then their decision should match their common initial preference.

In the Rarupongo Archipelago, the army belonging to the generals is spread out across n tiny islands, with one general on each island. The only means of communication is fireworks. A general on any island can shoot fireworks straight up into the air from his island. The fireworks can be seen anywhere within a radius of 30 kilometres. On each island, there are always islanders monitoring the sky in all directions for fireworks. Assume each island has an unlimited supply of fireworks. The algorithms followed by generals on different islands need not be identical.

Rarupongo is a very rainy place. Sometimes, when rainwater gets into a box of fireworks, the fireworks get damaged. Thus, when they are lit, they sometimes fizzle out without creating any visible signal. (This is considered a communication failure.)

Let G be a graph where each node represents an island. There is an edge connecting two nodes if and only if the two islands they represent are at most 30 kilometres apart. You may assume that all generals know the geography of their archipelago.

- (a) State a necessary and sufficient condition on the graph G for the problem to be solvable.
- (b) Show your condition is sufficient.

You can do this by giving an algorithm that works for every graph that satisfies the condition. You should give a detailed procedure for each general to follow that includes exactly how fireworks should be launched and how a general should decide on his attack time, based on the fireworks that were visible at his island. Briefly explain why your algorithm works.

- (c) Show your condition is necessary.

This means you should prove that, for all graphs that do not satisfy the condition, there is no algorithm that solves the problem.