EECS 3604 Lab 3 : Simulating the Wave Equation

Submission details: Write your responses to the following questions and submit them electronically as a lab report, along with any code that you write. If your responses are handwritten, scan them for electronic submission. Submission is via Moodle. Due date: March 14, 2016.

Grading details: 70% of your lab grade is for correctly completing the lab requirements; 20% is for clear writing and good presentation, including readable and well-documented code; 10% is for extra work or analysis that expands on or goes beyond the lab requirements.

Note: You may complete this lab in python (using numpy) instead of MATLAB if you wish.

For a function $u(t, x)$ of space and time, the one-dimensional wave equation is given by

$$\frac{\partial^2}{\partial x^2} u(t, x) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} u(t, x) = 0. \tag{1}$$

We will consider a simple, explicit numerical solution to the wave equation.

Let Δt and Δx represent small increments in time and space, respectively. A numerical approximation of the second derivatives in (1) is given by

$$\frac{\partial^2}{\partial x^2} u(t, x) \simeq \frac{u(t, x + \Delta x) - 2u(t, x) + u(t, x - \Delta x)}{\Delta x^2} \tag{2}$$

$$\frac{\partial^2}{\partial t^2} u(t, x) \simeq \frac{u(t + \Delta t, x) - 2u(t, x) + u(t - \Delta t, x)}{\Delta t^2} \tag{3}$$

Combining these approximations with (1), we have

$$\frac{u(t, x + \Delta x) - 2u(t, x) + u(t, x - \Delta x)}{\Delta x^2} = \frac{1}{c^2} \frac{u(t + \Delta t, x) - 2u(t, x) + u(t - \Delta t, x)}{\Delta t^2}. \tag{4}$$

Now we consider how to solve the approximate wave equation (4). We already know what happens in the present (at time t) and in the past (at time $t - \Delta t$). Thus, we are interested in what happens at the next time step, at $t + \Delta t$, and note that there is only one term in (4) that is a function of $t + \Delta t$.

Your tasks:

- Solve (4) for $u(t + \Delta t, x)$.
- Write a function in MATLAB, given a function $u(t, x)$ (where $u(t, x)$ is defined for a range of x and the times $\{t, t - \Delta t\}$), that returns $u(t + \Delta t, x)$. (Can you obtain $u(t + \Delta t, x)$ for the *entire* range of x? Why or why not?)
- Give several examples of inputs and outputs for your function.

Note that the wave velocity c and the step sizes Δt, Δx must be small in order for this method to work; otherwise, there are problems with the numerical stability of the algorithm.

Ideas for extra work portion.

- Create an animated movie in MATLAB showing the propagation of a wave, formed from solutions obtained by your algorithm.
- Write an "implicit" solution that avoids the stability problems of this version. For example see section 4.1.2.2 in:
  http://pauli.uni-muenster.de/tp/fileadmin/lehre/NumMethoden/WS0910/ScriptPDE/Wave.pdf