EECS 3604 Lab 1 : Numerical calculation of vector integrals

Submission details: Write your responses to the following questions and submit them electronically as a lab report, along with any code that you write. If your responses are handwritten, scan them for electronic submission. Submission is via Moodle. Due date: February 1, 2016.

Grading details: 70% of your lab grade is for correctly completing the lab requirements; 20% is for clear writing and good presentation, including readable and well-documented code; 10% is for extra work or analysis that expands on or goes beyond the lab requirements.

Note: You may complete this lab in python (using numpy) instead of MATLAB if you wish.

1. Write a MATLAB function, `pathIntegral(P)`, which provides a numerical calculation of the integral

$$\int_P (x^2 + y^2 + z^2) d\vec{\ell}. \tag{1}$$

   The path P is expressed as a $3 \times n$ array of points, where each column represents a single point. You may assume that the points in P are close enough together to approximate the differential $d\vec{\ell}$. The function returns a three-dimensional vector consisting of the x, y, and z components of the resulting vector. *Calculate this integral exactly for the path $P = (0, 0, 0) \rightarrow (1, 1, 1)$, and compare the result to the output of your function.* Show how the accuracy of the integration changes as the path step size changes. Also provide other numerical examples, not necessarily accompanied by the exact integral calculation.

2. Write a MATLAB function, `dotPathIntegral(P)`, which provides a numerical calculation of the integral

$$\int_P \Big((y + z)\hat{x} + (x + z)\hat{y} + (y + z)\hat{z} \Big) \cdot d\vec{\ell} \tag{2}$$

As in part 1, the path P is expressed as a $3 \times n$ array of points, where each column represents a single point. The function returns a scalar. *Calculate this integral exactly for the path $P = (0,0,0) \to (1,1,1)$, and compare the result to the output of your function.* Show how the accuracy of the integration changes as the path step size changes. Also provide other numerical examples, not necessarily accompanied by the exact integral calculation.

3. Write a MATLAB function, `volumeIntegral(V)`, which provides a numerical calculation of the integral

$$\int_V \hat{x}x + \hat{y}y + \hat{z}z^2 dV. \tag{3}$$

In this case, let the volume of integration is a cube: starting at the origin $(0,0,0)$, and ending at the point $V = (x,y,z)$ provided to the function. *Calculate this integral exactly for the cube with corners $V = (0,0,0) \to (1,1,1)$, and compare the result to the output of your function.* Show how the accuracy of the integration changes as the path step size changes. Also provide other numerical examples, not necessarily accompanied by the exact integral calculation.

Ideas for the extra work portion. Doing any one of the following would qualify you for the 10% "extra work" grade:

- Numerical calculation of vector derivatives for a given function, e.g. grad, div, curl
- Numerical calculation of a surface integral
- Numerical calculation for an *arbitrary* function over an *arbitrary* path, e.g. where the function is specified as points in a 3D space, and you must interpolate over the path
- Other components of comparable difficulty (ask the instructor if unsure)