Problem set 3

1. Consider the 1-d wave equation

$$\frac{\partial^2 V}{\partial x^2} - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = 0. \tag{1}$$

Let $f(\cdot)$ be any twice-differentiable function. Show that $V = f(t - x\sqrt{\mu\epsilon})$ is a solution of the wave equation.

2. Consider the wave equation

$$\nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = 0. \tag{2}$$

In spherical coordinates, assume the solutions are constant with respect to ϕ and θ, and let $V(r,t) = \frac{1}{r} U(r,t)$. With this substitution, show that the spherical wave equation reduces to

$$\frac{\partial^2 U(r,t)}{\partial r^2} - \mu\epsilon \frac{\partial^2 U(r,t)}{\partial t^2} = 0. \tag{3}$$

3. Show that

$$Q(x,y,z,t) = \cos(2\pi f t + 2\pi f x) + \cos(4\pi f t + 2\pi f y), \tag{4}$$

is *not* a solution to *any* wave equation, regardless of wave speed c.

4. Let

$$\vec{A}(x,t) = \cos(2\pi f t + \sqrt{\mu\epsilon} 2\pi f x)\hat{x}. \tag{5}$$

Using the Lorentz condition, find $V(x,t)$.