Test 2

First Name: ____________________________________

Last Name: ____________________________________

Student Number: __________________________

This test lasts 80 minutes. *No aids allowed.*
You may use any result that was proved in class or in the textbook without reproving it.
Make sure your test has 5 pages, including this cover page.
Answer in the space provided. (If you need more space, use the reverse side of the page and indicate ***clearly*** *which part of your work should be marked.)*
Write legibly.

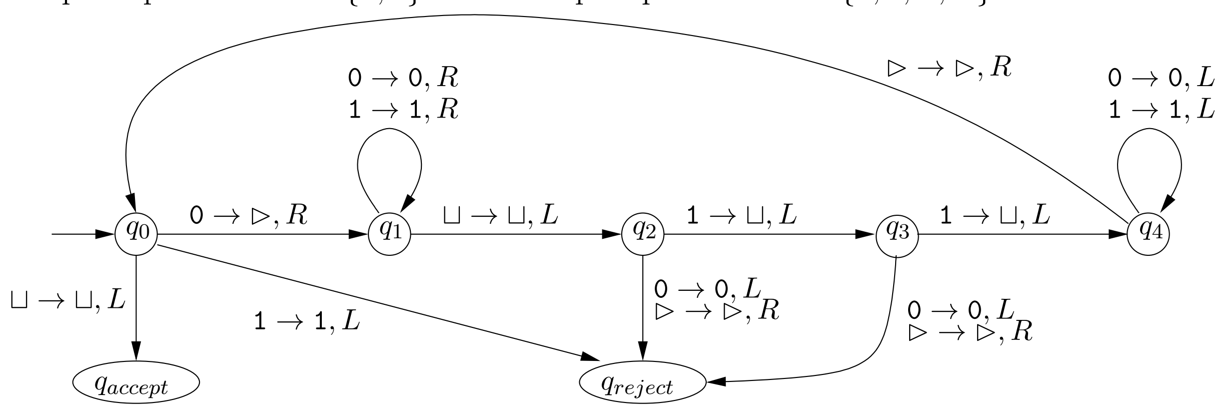
Question 1	/3
Question 2	/2
Question 3	/4
Question 4	/3
Question 5	/12
Total	/24

1. [3 marks] Give a precise definition of what it means for a language L to be decidable.

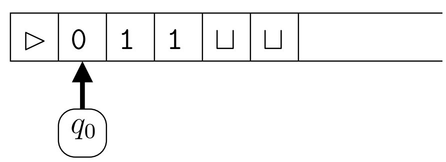
2. [2 marks] We saw in class that if L_1 and L_2 are recognizable languages, then $L_1 \cup L_2$ must also be recognizable. Explain *briefly* how this was proved.

3. [4 marks] Suppose M is a Turing machine that recognizes the language L. Let M' be the Turing machine that is identical to M, except the accept and reject states of M are swapped. Is it always true that M' recognizes $\overline{L}$? Explain why your answer is correct.

4. [3 marks] Consider the single-tape Turing machine whose tranisition diagram is shown below. The input alphabet is $\Sigma = \{0, 1\}$ and the tape alphabet is $\Gamma = \{0, 1, \sqcup, \triangleright\}$.



If the Turing machine is initially in the configuration below (with input string 011), show the configuration of the machine after each of its first five steps. For each configuration, show the tape contents, the head position and the state of the Turing machine.



Bonus question (worth 3 marks; attempt only if you have extra time): Describe exactly which strings are accepted by the Turing machine shown above.

5. [12 marks] In this question, we consider the problem of finding useless states in Turing machines (i.e., states that are impossible to reach).
Let $\text{USELESS}_{TM} = \{\langle M, q\rangle$: for *all* strings x, if the Turing machine M is run on input string x, it *never* reaches state $q\}$.

[1] **(a)** Describe precisely the property that machine M and state q satisfy when $\langle M, q\rangle$ is *not* in USELESS_{TM}.

[3] **(b)** Give an algorithm (in pseudocode) that recognizes $\overline{\text{USELESS}_{TM}}$. Explain why it is correct.

[4] **(c)** Prove that $\textsc{Useless}_{TM}$ is undecidable.

[2] **(d)** Is $\textsc{Useless}_{TM}$ recognizable? Prove your answer is correct.

[2] **(e)** After you have graduated from York, suppose you get a job working for a company that produces compilers. Your boss tells you to write a module for the compiler that takes a Java programme as input and removes all lines of the programme that can never be executed. How would you respond?