Test 1

First Name: ______________________________

Last Name: ______________________________

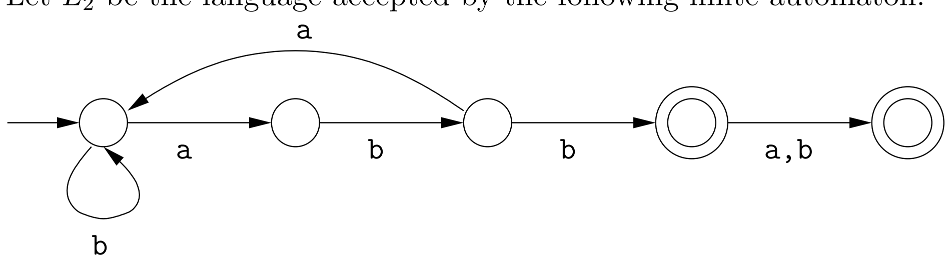
Student Number: ______________________

This test lasts 80 minutes. *No aids allowed.*
You may use any result that was proved in class or in the textbook without reproving it.
Make sure your test has 5 pages, including this cover page.
Answer in the space provided. (If you need more space, use the reverse side of the page and indicate ***clearly*** *which part of your work should be marked.)*
Write legibly.

Question 1	/4
Question 2	/3
Question 3	/3
Question 4	/4
Question 5	/4
Question 6	/3
Total	/21

[4] **1.** Let $L_1 = \{w \in \{\mathtt{a}, \mathtt{b}\}^* : w$ has an odd number of $\mathtt{a}$'s and each $\mathtt{a}$ is followed immediately by at least one $\mathtt{b}\}$. Draw the transition diagram of a deterministic finite automaton that accepts the language L_1. (You do not have to prove your answer is correct.)

[3] **2.** Let L_2 be the language accepted by the following finite automaton.



Give a regular expression for L_2. (You do not have to prove your answer is correct.)

[3] **3.** Complete the following definition. A nondeterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ is said to *accept* an input string w if and only if ...

[4] **4.** Let $L_4 = \{uu : u \in \{\texttt{0}, \texttt{1}\}^*\}$. For example, `0101101011` is in L_4 because it consists of two repetitions of `01011`. Prove that L_4 is not regular.

[4] **5.** Prove the following claim by induction on n.
Claim: For all $n \geq 0$, if L is a language that contains exactly n different strings, then L is regular.

[3] **6.** If $L \subseteq \Sigma^*$ is a language, let $SUPER(L)$ be the language of all strings that contain a string of L as a substring. More formally,

$$SUPER(L) = \{x \in \Sigma^* : \exists x_2 \in L, \exists x_1, x_3 \in \Sigma^* \text{ such that } x = x_1 x_2 x_3\}.$$

Prove that if L is regular, then $SUPER(L)$ is regular too.
(Hint: your answer can be quite short.)