Quiz

First Name: _____

Last Name: _____

Student Number: _____

*This test lasts **40 minutes**. No aids allowed.*
Make sure your test has 3 pages, including this cover page.
*Answer in the space provided. (If you need more space, use the reverse side of the page and indicate **clearly** which part of your work should be marked.)*
Write legibly.

Question 1	/3
Question 2	/2
Question 3	/3
Question 4	/7
Total	/15

[3] **1.** Let $A = \{n \in \mathbb{N} : 1 \leq n^2 \leq 10\}$ and $B = \{\texttt{ab}, \texttt{d}\}$. Give an explicit listing of the all elements of the following sets.

(a) $A \cup B =$

(b) $A \times B =$

(c) The power set of $B =$

[2] **2.** Are the following statements true or false? Assume the domain of x and y is the set of all positive real numbers. (You do not have to prove your answers are correct.)

(a) $\forall x, \exists y$ such that $x = y^2$

(b) $\exists y$ such that $\forall x, x = y^2$

[3] **3.** Let $L = \{s \in \{\texttt{0}, \texttt{1}\}^* : s$ does not contain `110` as a substring$\}$. Draw the transition diagram for a (deterministic) finite automaton for the language L. You do not have to prove your answer is correct.

[7] **4.** We define a sequence $s_0, s_1, s_2, \ldots$ of strings over the alphabet $\{\mathtt{a}, \mathtt{b}\}$ recursively as follows.

$$\begin{aligned} s_0 &= \mathtt{b} \\ s_1 &= \mathtt{a} \\ s_k &= s_{k-2}\mathtt{a}s_{k-1}\mathtt{a}s_{k-2}, \text{ for } k \geq 2 \end{aligned}$$

(a) Write down the string s_3.

(b) Fill in the blank in the following claim with a numerical expression involving n.

Claim: For all $k \geq 0$, the number of $\mathtt{a}$'s in s_k is _____.

(c) Give a careful proof of the claim in part (b).