Random Access Techniques: ALOHA (cont.)

**Example**  [Aloha – avoiding collision]

A pure ALOHA network transmits a 200-bit frame on a shared channel Of 200 kbps at time $t_0$. What is the requirement to make this frame collision free?

**Solution:**

Frame transmission time $T_f = \frac{200 \text{ bits}}{2000 \text{ kbps}} = 1 \text{ msec.}$

Vulnerability period = $2 \times 1 \text{ msec} = 2 \text{ msec}$

Collision will be avoided if no other station start transmitting 1 msec before and during the transmission of this frame.
**Throughput**

- **Definitions and Assumptions:**
  - $S$ ($S_{out}$) – **throughput**: average # of successful frame transmission per $X$ sec (if network operates under ‘stable conditions’ $S_{out} = S_{in}$, where $S_{in}$ - arrival rate of new frames to the system)
  - $G$ – load – average # of overall transmission attempts per $X$ sec
  - $P_{succ}$ – probability of a successful frame transmission

\[ S = P_{succ} \cdot G \]

- **How to find $P_{succ}$?** – suppose a frame is **ready for transmission** at time $t_0$ – frame will be transmitted successfully if no other frame attempts transmission $X$ sec before and after $t_0$

- random backoff spreads retransmissions so that frame transmission (arrivals) are equally likely at any instant in time – Poisson process!!!
if general, if frame arrivals are equally likely at any instant in time, and arrivals occur at an average rate of $\lambda$ [arrivals per sec]

• Poisson process

\[ P[k \text{ arrivals in } T \text{ seconds}] = \frac{(\lambda T)^k}{k!} e^{-\lambda T} \]

• to get [arrivals per second] $\lambda$ is calculated as $\lambda = G/X$, while interval of interest is $T = 2X$, hence

\[ P[k \text{ transmissions in } 2X \text{ seconds}] = \frac{(2G)^k}{k!} e^{-2G} \]

• thus, probability of successful transmission (no other transmission in $T = 2X$ seconds) is:

\[ P_{\text{succ}} = P[0 \text{ transmissions in } 2X \text{ seconds}] = e^{-2G} \]

and throughput

\[ S = G \cdot P_{\text{succ}} = G \cdot e^{-2G} \]
Random Access Techniques: ALOHA (cont.)

S vs. G in Pure ALOHA

- NOTE: our analysis assumed that many nodes share a common channel & have comparable transmiss. rates (if only 1 node uses the medium, S=1)

- initially, as G increases S increases until it reaches $S_{max}$ – after that point the network enters ‘unstable operating conditions’ in which collisions become more likely and the number of backlogged stations increases (consequently, $S_{in} > S_{out}$)

- max throughput of ALOHA ($S_{max} = 0.184$) occurs at $G=0.5$, which corresponds to a total arrival rate of ‘one frame per vulnerable period’

- $S_{max} = 0.184 \Rightarrow$ max ALOHA throughput = 18% of channel capacity

18% of channel utilization, with Aloha, is not encouraging. But, with everyone transmitting at will we could hardly expect a 100% success rate.
Example [Aloha]

A pure ALOHA network transmits 200-bits frames on a shared channel of 200 kbps.

What is the throughput of this system if all stations together produce 1000 frames per second?

Solution:

Throughput of pure ALOHA:

\[ S = G \cdot e^{-2G} \]  \[ \text{frames} \]  \[ \text{frame time} \]

\[ G = 1000 \]  \[ \text{frames / second} \], but we need it in  \[ \text{frames / frame time} \]

Frame time \[ X = 200 \] bits / 200 kbps = 1 msec

\[ G = 1000 \]  \[ \text{frames / 1000 frame times} \] = 1 [frame / frame time]

Hence,

\[ S(G = 1) = e^{-2} = 0.135 \]  \[ \text{frames} \]  \[ \text{frame time} \] = 0.135  \[ \text{frames} \]  \[ 1 \text{msec} \] = 135  \[ \text{frames} \]  \[ \text{sec} \]
Example  [ Aloha ]

a) What is the vulnerable period (in milliseconds) of a pure ALOHA broadcast system with \( R = 50 \text{ kbps} \) wireless channel, assuming 1000-byte frames.

\[
X = \frac{1000 \text{ bytes}}{50 \text{ kbps}} = \frac{8000 \text{ bits}}{50 \text{ kbps}}
\]

\[
X = 160 \text{ milliseconds}
\]

vulnerable period = \( 2 \times X = 320 \text{ milliseconds} \)

b) What is the maximum possible throughput \( S \) of such a channel (system), in kbps?

\[
S = 0.18 \times R = 0.18 \times 50 \text{ kbps} = 9.179 \text{ kbps}
\]

b) We do not have enough information to determine \( G \). The best we can do ...
Slotted ALOHA – “improved ALOHA”, with reduced probability of collision

- assumptions:
  - time is divided into slots of size $X=L/R$ (one frame time)
  - nodes start to transmit only at the beginning of a slot
  - nodes are synchronized so that each node knows when the slots begin

- operation:
  1) when node has a fresh frame to send, it waits until next frame slot and transmits
  3) if there is a collision, node retransmits the frame after a backoff-time (backoff-time = multiples of time-frames)
Example [ Aloha vs. Slotted Aloha ]

Random Access Techniques: Slotted ALOHA (cont.)
Vulnerable Period of Slotted ALOHA

- consider one arbitrary packet P that becomes ready for transmission at some time t during the time slot \([k, k+1]\)

- packet P will be transmitted successfully if no other packet becomes available for transmission during the same time slot

\[
\text{vulnerable period } = [ t_0 - X, t_0 ]
\]

\[
P_{\text{succ}} = P[0 \text{ arrivals in } X \text{ seconds}] = e^{-G}
\]

\[
S = G \cdot P_{\text{succ}} = G \cdot e^{-G}
\]
**S vs. G in Slotted ALOHA**

- max throughput of Slotted ALOHA ($S_{\text{max}} = 0.36$) occurs at $G=1$, which corresponds to a total arrival rate of ‘one frame per vulnerable period’
- $S_{\text{max}} = 0.36 \Rightarrow$ max Slotted ALOHA throughput = 36% of actual channel capacity

**Slotted ALOHA vs. Pure ALOHA**

- slotted ALOHA reduces vulnerability to collision, but also adds a waiting period for transmission
- if contention is low, it will prevent very few collisions, & delay many of the (few) packets that are sent
Example [ slotted Aloha ]

Measurements of slotted ALOHA channel with an infinite number of users show that 10% of the slots are idle.

a) What is the channel load, $G$?
b) What is the throughput, $S$?
c) Is the channel underloaded or overloaded?

\[ a) \text{ 10\% of slots idle } \Rightarrow \text{ frame will be successfully transmitted if sent in those 10\% of slots } \Rightarrow \text{ } P_{\text{succ}} = 0.1 \]
\[ \text{According to theory, } P_{\text{succ}} = e^{-G} \Rightarrow G = -\ln(P_{\text{succ}}) = -\ln(0.1) = 2.3 \]

\[ b) \text{ According to theory, } S = P_{\text{succ}} \times G = G \times e^{-G} \]
\[ \text{as } G=2.3 \text{ and } e^{-G}=0.1 \Rightarrow S = 0.23 \]

\[ c) \text{ Whenever } G>1, \text{ the channel is overloaded, so it is overloaded in this case.} \]
Example  [ slotted Aloha in cellular (GSM) systems ]
• if general, if frame arrivals are equally likely at any instant in time, and arrivals occur at an average rate of $\lambda$ [arrivals per sec]

\[ P[k \text{ arrivals in } T \text{ seconds}] = \frac{\left(\frac{\lambda T}{k!}\right)^k e^{-\lambda T}}{k!} \]

Poison process

• to get [arrivals per second], $\lambda$ is calculated as $\lambda = \frac{G}{X}$

• to avoid collision, there should be only 1 transmission in the interval $X$ starting at $t_0$ AND 0 transmissions in the interval $X$ preceding $t_0$

\[ P_1 = P[\text{1 transmission in } X \text{ seconds}] = \frac{\left(\frac{G}{X}\right)^1}{1!} e^{-\frac{G}{X}} = \frac{G}{X} e^{-G} \]

\[ P_2 = P[\text{0 transmissions in } X \text{ seconds}] = e^{-G} \]

\[ P_{\text{succ}} = P_1 \cdot P_2 = G \cdot e^{-2G} \]