Data Rate Limits in Digital Transmission

Max Data Rate [bps] Over a Channel? – depends on three factors:

- bandwidth available
- # of levels in digital signal
- quality of channel – level of noise

Nyquist Theorem – defines theoretical max bit rate in noiseless channel [1924]
- even perfect (noiseless) channels have limited capacity

Shannon Theorem – Nyquist Theorem extended - defines theoretical max bit rate in noisy channel [1949]
- if random noise is present, situation deteriorates rapidly!
**Intersymbol Interference** – the inevitable filtering effect of any practical channel will cause spreading of individual data symbols that pass through the channel

- this spreading causes part of symbol energy to overlap with neighbouring symbols causing **intersymbol interference (ISI)**
- ISI can significantly degrade the ability of the data detector to differentiate a current symbol from the diffused energy of the adjacent symbols

As the channel bandwidth $B$ increases, the width of the impulse response decreases $\Rightarrow$ pulses can be input in the system more closely spaced, i.e. at a higher rate.

$$T_s = \frac{1}{2B}$$
**Impulse Response** — response of a low-pass channel (of bandwidth $B$) to a narrow pulse $h(t)$, aka Nyquist pulse:

$$s(t) = \frac{\sin(2\pi B t)}{2\pi B t}$$

- zeros: where $\sin(2\pi B t) = 0 \Rightarrow t = k \cdot \frac{1}{2B}, \quad k = 1, 2, 3, ...$

What is the minimum pulse/bit duration time to avoid significant ISI?!
Example  [ problems associated with intersymbol interference ]
Example  [ system response to binary input 110 ]

Assume:  channel bandwidth = max analog frequency passed = B [Hz].

New pulse is sent every $T_S$ sec ⇒ data rate = $1/T_S$ [bps] = 2B [bps]

The combined signal has the correct values at $t = 0, 1, 2$.

$$r_{max} = \frac{1 \text{ pulse}}{T_S \text{ second}} = 2W = 2B \left[ \frac{\text{pulses}}{\text{second}} \right]$$

Maximum signaling rate that is achievable through an ideal low-pass channel.
Nyquist Law — max rate at which digital data can be transmitted over a communication channel of bandwidth $B$ [Hz] is

$$C_{\text{noiseless}} = 2 \cdot B \cdot \log_2 M \text{ [bps]}$$

- $M$ — number of discrete levels in digital signal
- $M \uparrow \Rightarrow C \uparrow$, however this places increased burden on receiver — instead of distinguishing one of two possible signals, now it must distinguish between $M$ possible signals
  - especially complex in the presence of noise

If spacing between levels becomes too small, noise signal can cause receiver to make wrong decision.
Example  [ multilevel digital transmission ]

2-level encoding:  $C = 2B$ [bps]
one pulse – one bit

4-level encoding:  $C = 2 \times 2 = 4B$ [bps]
one pulse – two bits

8-level encoding:  $C = 2 \times 3 = 6B$ [bps]
one pulse – three bits

Data Rate Limits: Nyquist Theorem
**Shannon Law** – maximum transmission rate over a channel with bandwidth $B$, with Gaussian distributed noise, and with signal-to-noise ratio $\text{SNR}=S/N$, is

$$C_{\text{noisy}} = B \cdot \log_2 (1 + \text{SNR}) \text{ [bps]}$$

- **theoretical limit** – there are numerous impairments in every real channel besides those taken into account in Shannon's Law (e.g. attenuation, delay distortion, or impulse noise)

- **no indication of levels** – no matter how many levels we use, we cannot achieve a data rate higher than the capacity of the channel

- in practice we need to use both methods (Nyquist & Shannon) to find what data rate and signal levels are appropriate for each particular channel:

The Shannon capacity gives us the upper limit! The Nyquist formula tells us how many levels we need!
Example [ data rate over telephone line ]

What is the theoretical highest bit rate of a regular telephone line?
A telephone line normally has a bandwidth of 3000 Hz (300 Hz to 3300 Hz). The signal-to-noise ratio is usually 35 dB (3162) on up-link channel (user-to-network).

Solution:
We can calculate the theoretical highest bit rate of a regular telephone line as

\[ C = B \log_2 (1 + SNR) = \]
\[ = 3000 \log_2 (1 + 3162) = \]
\[ = 3000 \log_2 (3163) \]

\[ C = 3000 \times 11.62 = 34,860 \text{ bps} \]
**Example**  [ data rate / number of levels ]

We have a channel with a 1 MHz bandwidth. The SNR for this channel is 63; what is the appropriate bit rate and number of signal level?

**Solution:**

First use Shannon formula to find the upper limit on the channel’s data-rate

\[
C = B \log_2 (1 + \text{SNR}) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 (64) = 6 \text{ Mbps}
\]

Although the Shannon formula gives us 6 Mbps, this is the upper limit. For better performance choose something lower, e.g. 4 Mbps.

Then use the Nyquist formula to find the number of signal levels.

\[
C = 2 \cdot B \cdot \log_2 M \ [\text{bps}] \\
4 \text{ Mbps} = 2 \times 1 \text{ MHz} \times \log_2 L \quad \Rightarrow \quad L = 4
\]