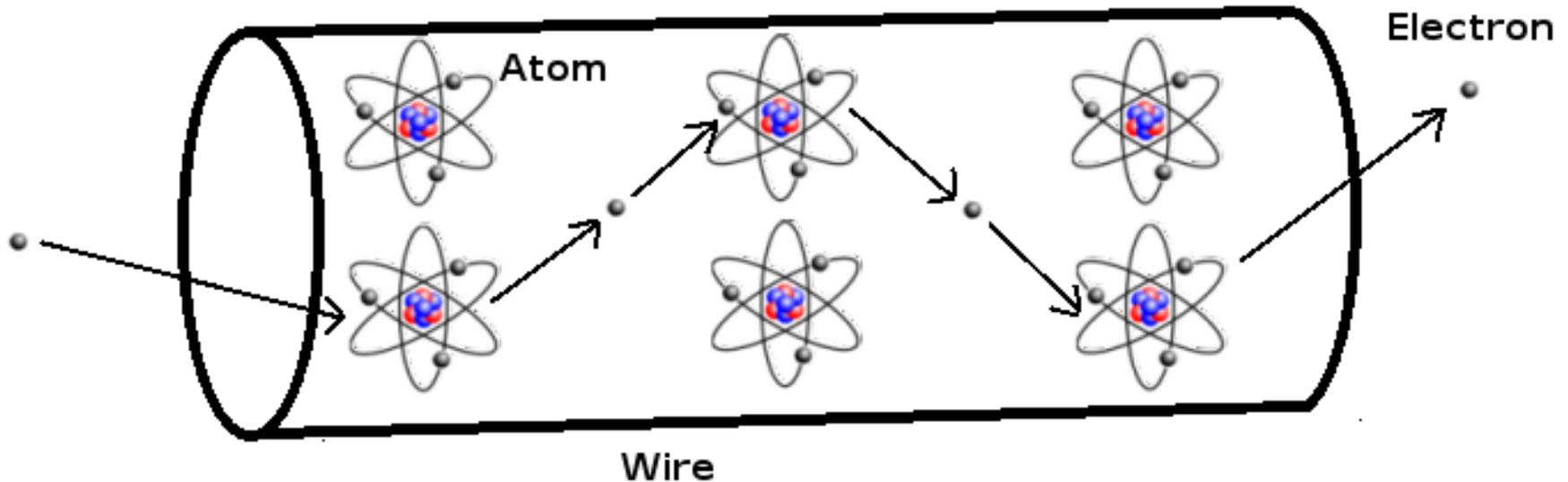


Data vs. Signal (cont.)

Flow of Electrons Through a Conductor ...



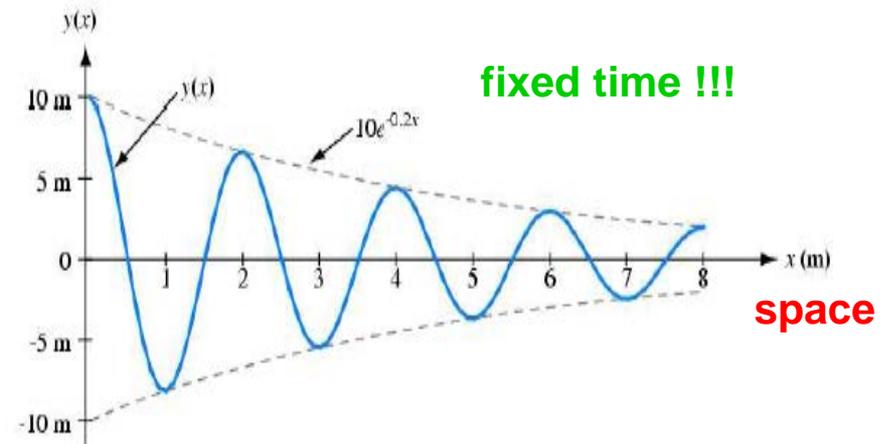
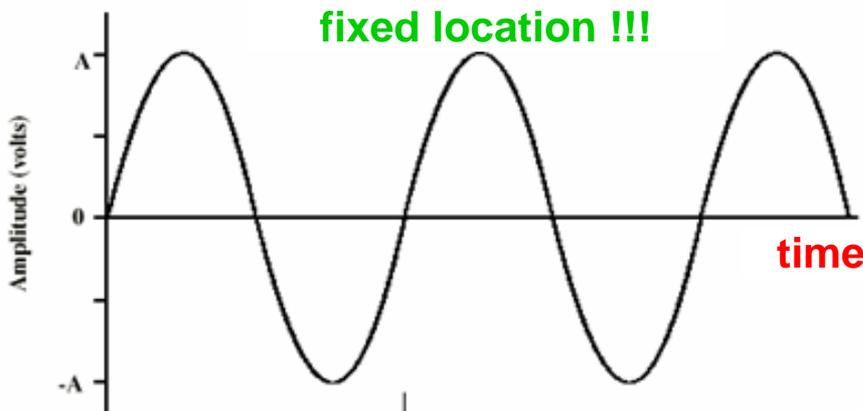
<https://electronicspani.com/what-is-electric-current/>

Signal Representation

Signal Representation – typically in 2D space, as a function of time, space or frequency



- when horizontal axis is time, graph displays the value of a signal at one particular point in space as a function of time
- when horizontal axis is space, graph displays the value of a signal at one particular point in time as a function of space



The time- and space- representation of a signal often resemble each other, though the signal envelope in the space-representation is different (signal attenuates over distance).

- **Data vs. Signal**
- **Analog vs. Digital**
- **Analog Signals**
 - Simple Analog Signals
 - Composite Analog Signals
- **Digital Signals**

Analog vs. Digital Data

- **analog data** – representation variable takes on continuous values in some interval, e.g. **voice, temperature**, etc. recording/reading
- **digital data** – representation variable takes on discrete (a finite & countable number of) values in a given interval, e.g. **text, digitized images**, etc.

Analog vs. Digital Signal

- **analog signal** – signal that is continuous in time and can assume an infinite number of values in a given range (continuous in time and value)
- **discrete (digital) signal** – signal that is continuous in time and assumes only a limited number of values (maintains a constant level and then changes to another constant level)

Stallings, pp. 65 “This is an idealized definition. In fact, the transition from one voltage to another is not instantaneous, but there is a small transition period.”

Analog vs. digital data ...

IMAGES

ANALOG



Continuous



For Human Viewing

DIGITAL



Matrix of Pixels

Each Image Point

56	56	57	56
56	56	57	56
57	57	57	59
58	58	58	60

For Computer Systems

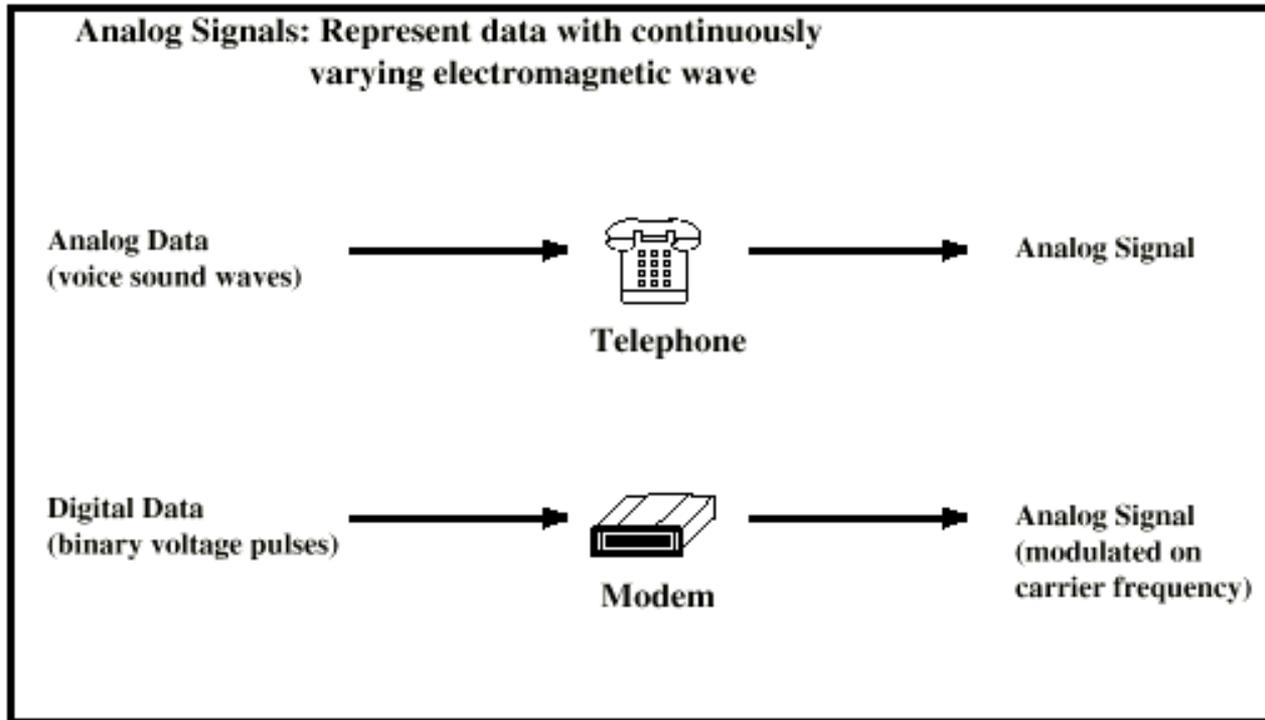


Brightness
Film Density
Color

Number

Both analog and digital data can be transmitted using either analog or digital signals.

DATA



SIGNAL



example: analog signaling of analog and digital data

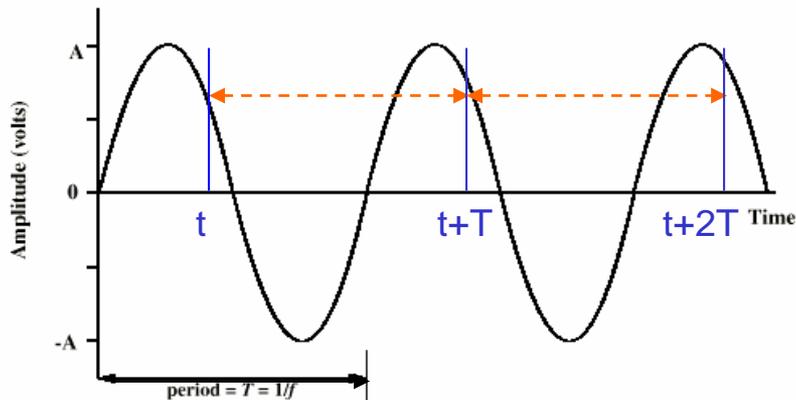
... will talk more about this later ...

Periodic vs. Aperiodic Signals

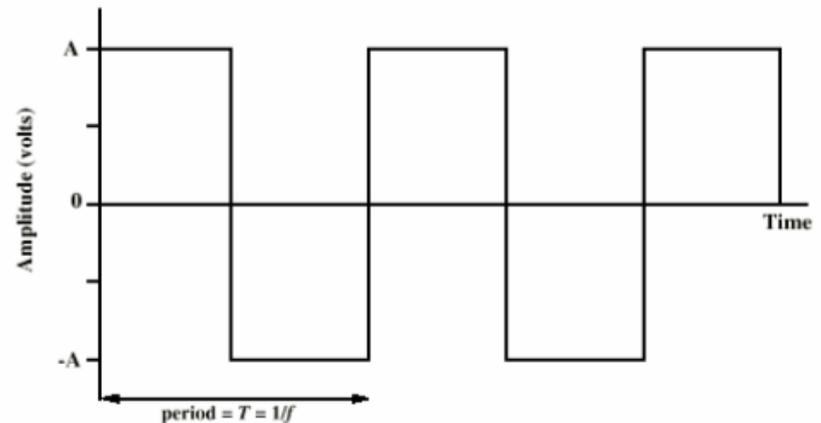
- **periodic signal** – completes a pattern within some measurable time frame, called a **period (T)**, and then repeats that pattern over subsequent identical periods

$$\exists T \in \mathbb{R} \text{ s. t. } s(t+T) = s(t), \forall t \in \langle -\infty, +\infty \rangle$$

- T - smallest value that satisfies the equation
- T is (typically) expressed in seconds
- **aperiodic signal** – changes without exhibiting a pattern that repeats over time

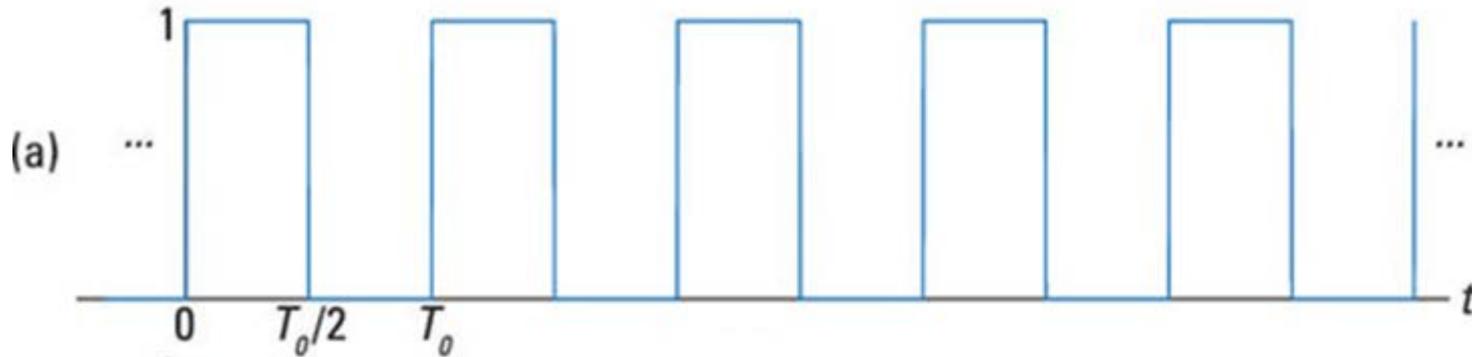


periodic analog signal

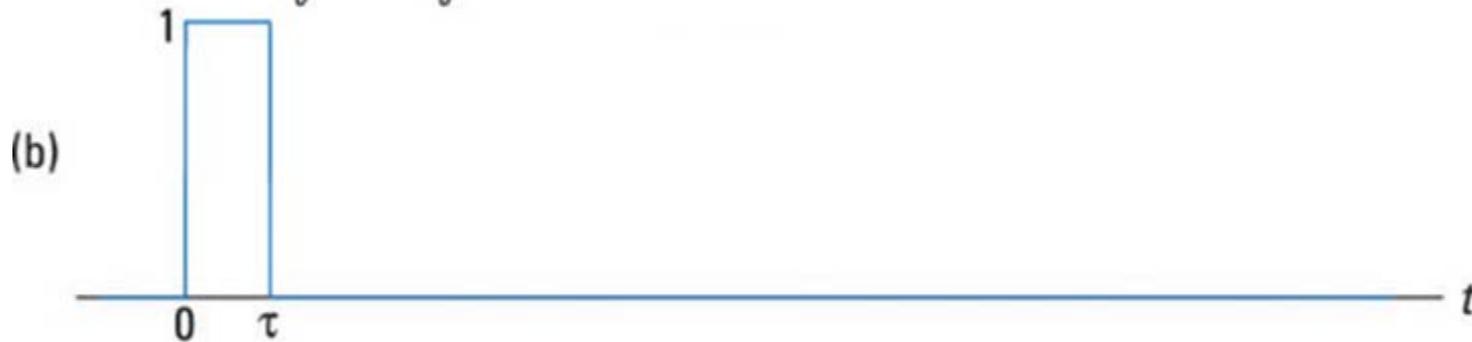


periodic digital signal

Different types of signals (analog, digital, periodic, aperiodic, ...)



periodic &
digital



aperiodic &
digital &
deterministic!



aperiodic &
analog &
random!

- **Data vs. Signal**
- **Analog vs. Digital**
- **Analog Signals**
 - Simple Analog Signals
 - Composite Analog Signals
- **Digital Signals**

Analog Signals

Classification of Analog Signals

(1) **Simple Analog Signal** – cannot be decomposed into simpler signals

- **sinewave** – most fundamental form of periodic analog signal – mathematically described with 3 parameters

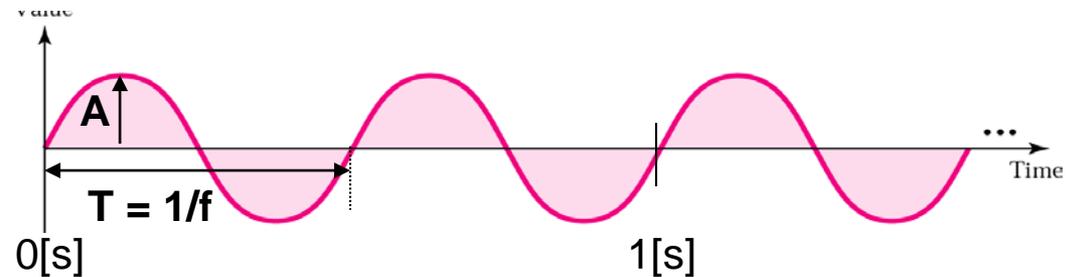
$$s(t) = A \cdot \sin(2\pi ft + \varphi)$$

(1.1) **peak amplitude (A)** – absolute value of signal's highest intensity – unit: **volts [V]**

(1.2) **frequency (f)** – number of periods in one second – unit: **hertz [Hz] = [1/s]** – inverse of period (T)!

(1.3) **phase (φ)** – absolute position of the waveform relative to an arbitrary origin – unit: **degrees [°] or radians [rad]**

The origin is usually taken as the last previous passage through zero from the negative to the positive direction.



(2) **Composite Analog Signal** – composed of multiple sinewaves

Simple Analog Signals

Sine wave pattern occurs often in nature, including ocean waves, sound waves, and light waves.



The oscillation of an undamped spring-mass system around the equilibrium is a sine wave

2.3.4 Simple Harmonic Motion (SHM)

Sinusoidal waves are not mathematical abstractions, they can be produced in the real world by a phenomenon called **Simple Harmonic Motion (SHM)**. An SHM is a specific type of oscillation exhibited by the spring loaded weight in Fig. 2.7. Assume that point P is rotating along a circle with uniform speed. A spring loaded weight W is attached to the point P with a flexible string T . As P moves along the circle in the anti-clockwise direction, the weight W attached to P oscillates up and down in synchronism. If a pen is attached to the backside of W which marks on a paper moving slowly to the right to represent the flow of time, the pen would trace out a sinusoidal curve. If ω is the angular frequency of the point P in radian/sec, then the frequency of the generated wave is given by:

$$f = \omega/2\pi$$

where f is in cycles/sec. This is apparent from Fig. 2.7, because as a complete cycle is traced out by the weight W , the point P moves by 2π radians, i.e. a complete circle. In 1 second since f cycles are traced out, the points moves a total of $2\pi f$ radians, which by definition is the angular frequency ω

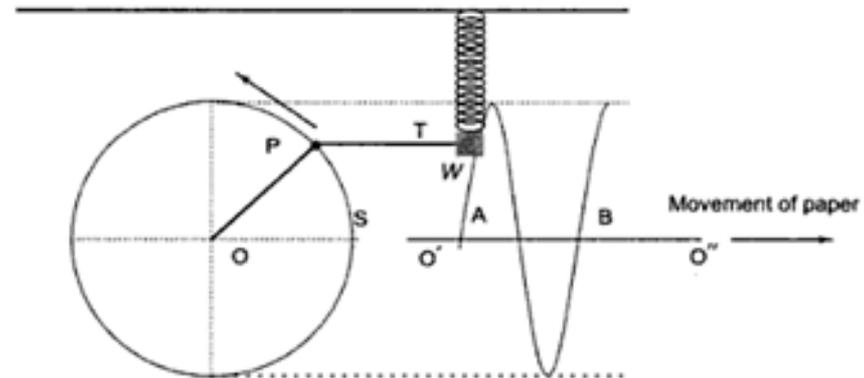


Figure 2.7

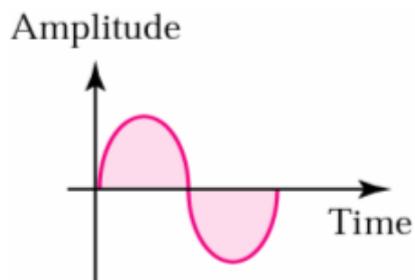
Simple Harmonic Motion (SHM)

Simple Analog Signals

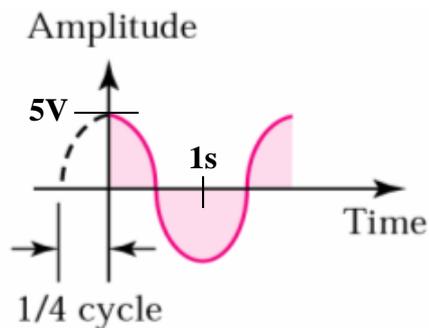
Phase in Simple Analog Signals

– measured in **degrees** or **radians**

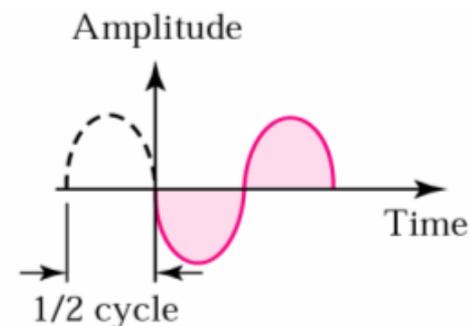
- $360^\circ = 2\pi$ rad
- $1^\circ = 2\pi/360$ rad
- 1 rad = $(360/2\pi)^\circ = 57.29578^\circ$
- phase shift of $360^\circ =$ shift of 1 complete period
- phase shift of $180^\circ =$ shift of 1/2 period
- phase shift of $90^\circ =$ shift of 1/4 period



$$\varphi = 0^\circ \text{ or } 360^\circ$$



$$\varphi = 90^\circ$$



$$\varphi = 180^\circ$$

Example [period (T) and frequency (f)]

Unit	Equivalent	Unit	Equivalent
seconds (s)	1 s	hertz (Hz)	1 Hz
milliseconds (ms)	10^{-3} s	kilohertz (KHz)	10^3 Hz
microseconds (μ s)	10^{-6} s	megahertz (MHz)	10^6 Hz
nanoseconds (ns)	10^{-9} s	gigahertz (GHz)	10^9 Hz
picoseconds (ps)	10^{-12} s	terahertz (THz)	10^{12} Hz

units of period and respective frequency

(a) Express a period of 100 ms in microseconds.

$$100 \text{ ms} = 100 \times 10^{-3} \text{ s} = 100 \times 10^{-3} \times 10^6 \mu\text{s} = 10^5 \mu\text{s}$$

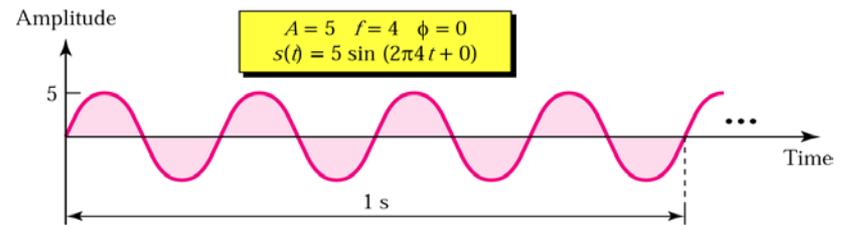
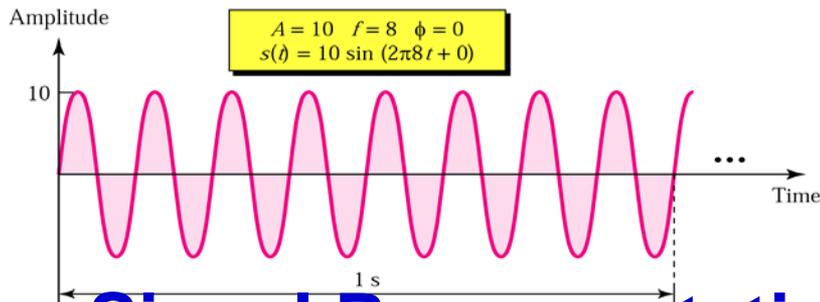
(b) Express the corresponding frequency in kilohertz.

$$100 \text{ ms} = 100 \times 10^{-3} \text{ s} = 10^{-1} \text{ s}$$

$$f = 1/10^{-1} \text{ Hz} = 10 \times 10^{-3} \text{ KHz} = 10^{-2} \text{ KHz}$$

Frequency in Simple Analog Signals – rate of signal change with respect to time

- change in a short span of time \Rightarrow $f=??$
- change over a long span of time \Rightarrow $f=??$
- signal does not change at all \Rightarrow $f=??$
??
- signal changes instantaneously \Rightarrow $f=??$
??



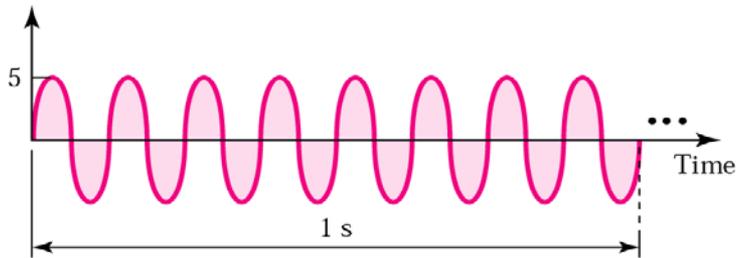
Signal Representation in Frequency Domain

Time Domain Plot – specifies signal amplitude at each instant of time

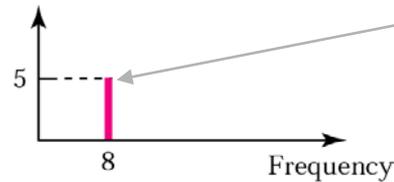
- does NOT express explicitly signal's phase and frequency

Frequency Domain Plot – specifies peak amplitude with respect to freq.

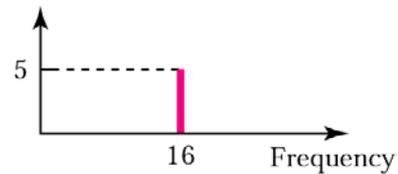
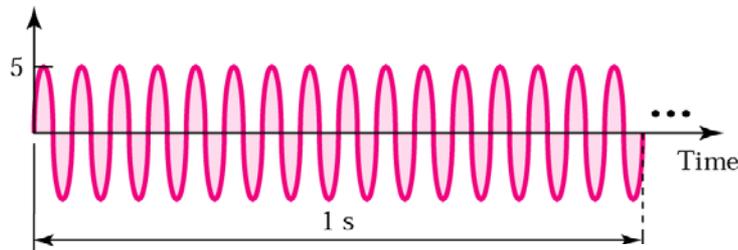
- phase CANNOT be shown in the frequency domain



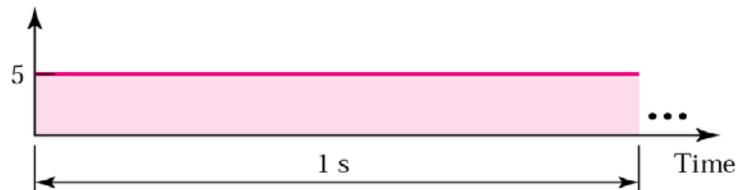
b. A signal with frequency 8



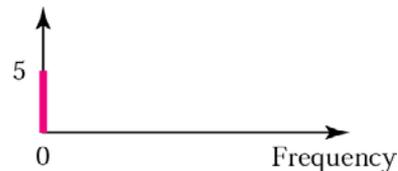
One 'spike' in frequency domain shows two characteristics of the signal:
spike position = signal frequency,
spike height = peak amplitude.



Time domain

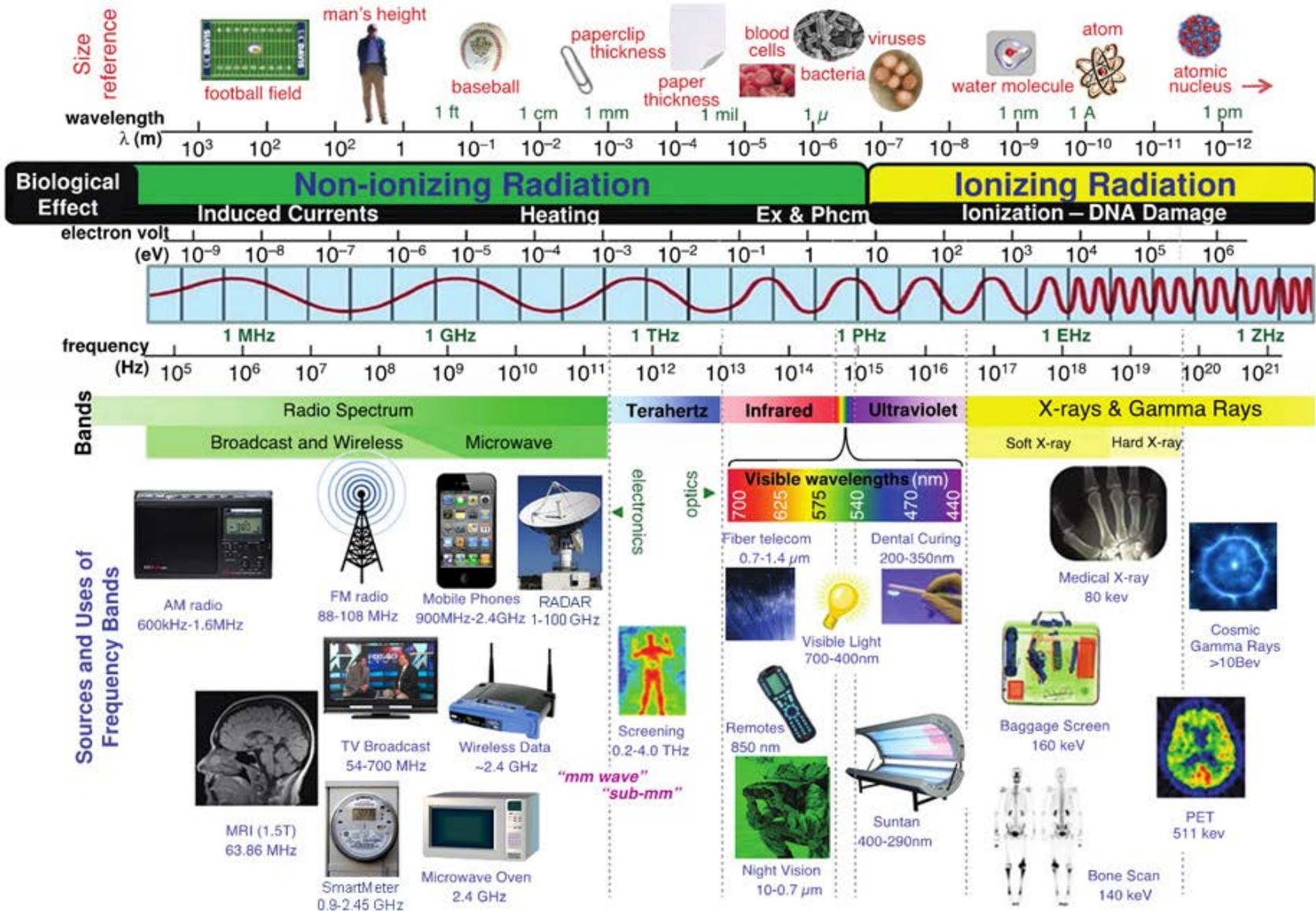


Frequency domain



Analog signals are best represented in the frequency domain.

ELECTROMAGNETIC RADIATION SPECTRUM



http://earthguide.ucsd.edu/eoc/special_topics/teach/sp_climate_change/p_emspectrum_interactive.html

<http://www-tc.pbs.org/wgbh/nova/assets/swf/1/electromagnetic-spectrum/electromagnetic-spectrum.swf>

Composite Analog Signals

Fourier Analysis – any composite signal can be represented as a **combination of simple sine waves** with different frequencies, phases and amplitudes

$$s(t) = A_1 \sin(2\pi f_1 t + \varphi_1) + A_2 \sin(2\pi f_2 t + \varphi_2) + \dots$$

- periodic composite signal (**period=T, freq.= $f_0=1/T$**) can be represented as a sum of simple sines and/or cosines known as *Fourier series*:

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t)]$$

With the aid of good table of integrals, it is easy to determine the frequency-domain nature of many signals.



$$A_n = \frac{2}{T} \int_0^T s(t) \cos(2\pi n f_0 t) dt, \quad n = 0, 1, 2, \dots$$

$$B_n = \frac{2}{T} \int_0^T s(t) \sin(2\pi n f_0 t) dt, \quad n = 1, 2, 3, \dots$$

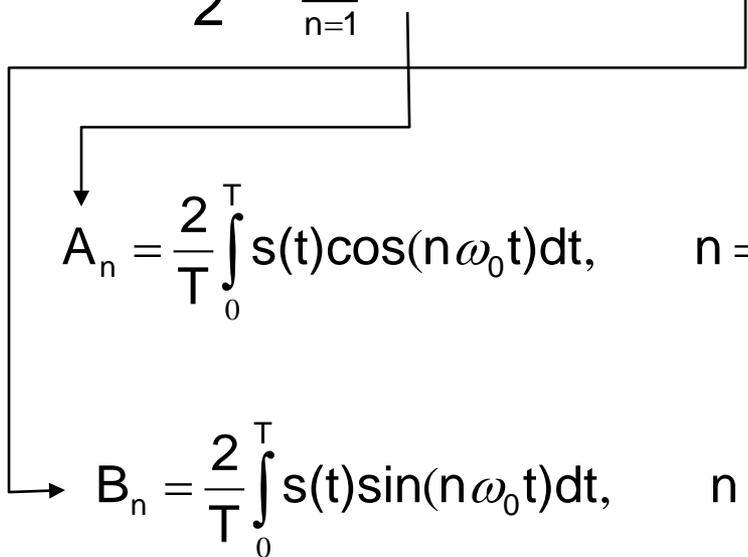
- f_0 is referred to as 'fundamental frequency'
- integer multiples of f_0 are referred to as harmonics

Angular Frequency – aka radian frequency – number of 2π revolutions during a single period of a given signal

$$\omega = \frac{2\pi}{T} = 2\pi \cdot f$$

- **simple multiple of ordinary angular frequency**

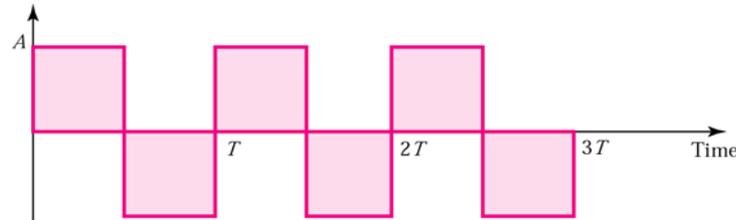
$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t)]$$


$$A_n = \frac{2}{T} \int_0^T s(t) \cos(n\omega_0 t) dt, \quad n = 0, 1, 2, \dots$$

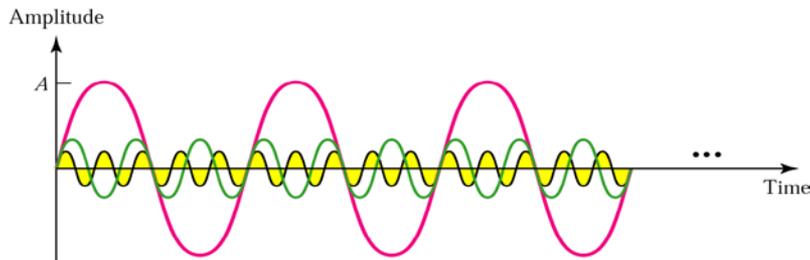
$$B_n = \frac{2}{T} \int_0^T s(t) \sin(n\omega_0 t) dt, \quad n = 1, 2, \dots$$

Example [periodic square wave]

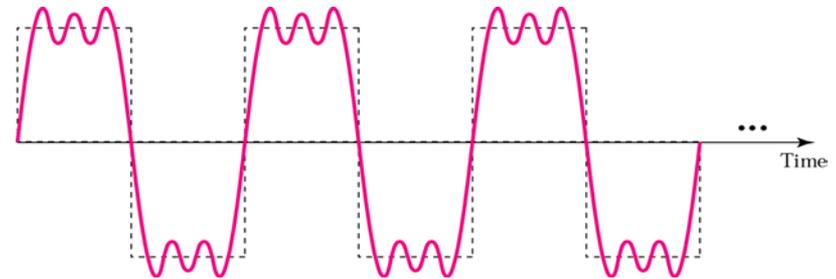
No DC component!!!



$$s(t) = \frac{4A}{\pi} \sin(2\pi ft) + \frac{4A}{3\pi} \sin(2\pi(3f)t) + \frac{4A}{5\pi} \sin(2\pi(5f)t) + \dots$$



three harmonics



adding three harmonics

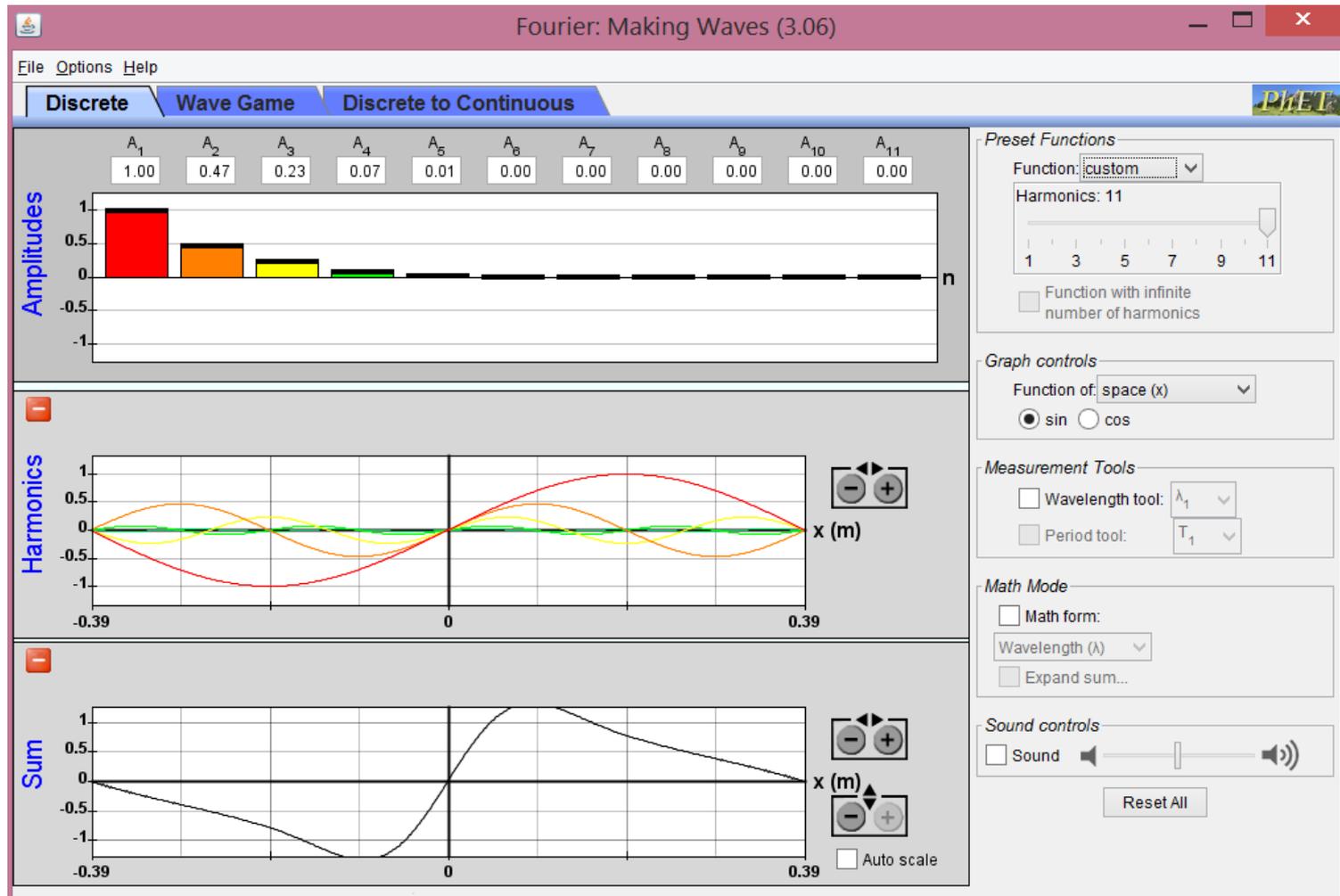
**With three harmonics we get an approximation of a square wave.
To get the actual square, all harmonics up to ∞ should be added.**

https://en.wikipedia.org/wiki/File:Fourier_series_square_wave_circles_animation.gif

<http://www.acs.psu.edu/drussell/Demos/Fourier/Fourier.html>

<http://www.nickkasprak.com/blog/fourier-series-animator>

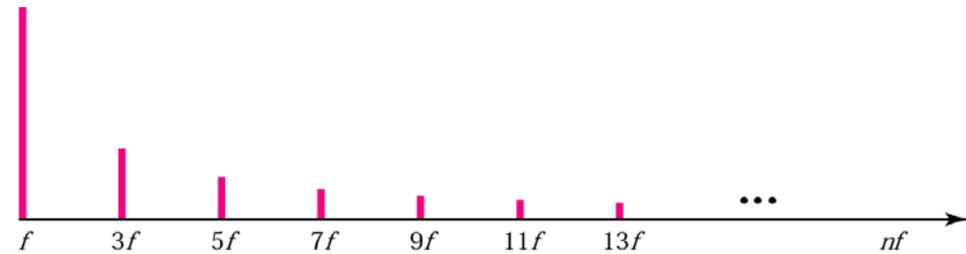
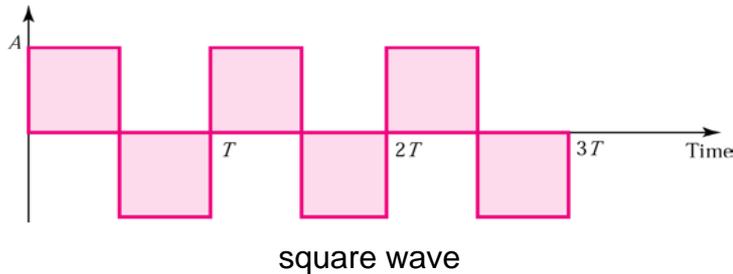
Example [composite analog signal]



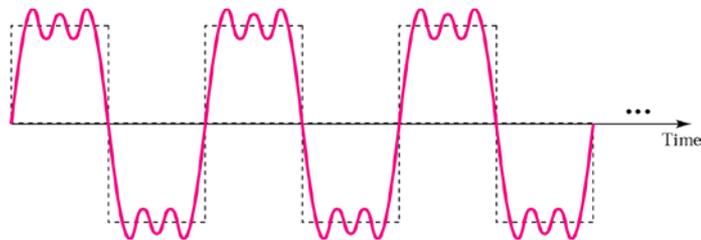
Frequency Spectrum – range (set) of frequencies that signal contains of Analog Signal

Absolute Bandwidth – width of signal spectrum: $B = f_{\text{highest}} - f_{\text{lowest}}$ of Analog Signal

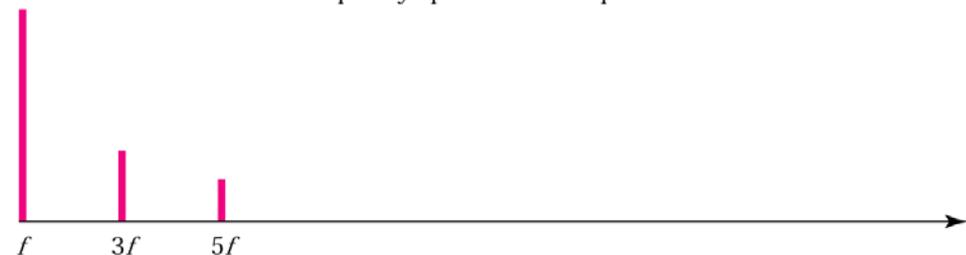
Effective Bandwidth – range of frequencies where signal contains most of its power/energy of Analog Signal



a. Frequency spectrum of a square wave



3-harmonic representation



b. Frequency spectrum of an approximation with only three harmonics

Example [frequency spectrum and bandwidth of analog signal]

A periodic signal is composed of five sinewaves with frequencies of 100, 300, 500, 700 and 900 Hz.

What is the **bandwidth** of this signal?

Draw the **frequency spectrum**, assuming all components have a max amplitude of 10V.

Solution:

$$B = f_{\text{highest}} - f_{\text{lowest}} = 900 - 100 = 800 \text{ Hz}$$

The spectrum has only five spikes, at 100, 300, 500, 700, and 900.

