

### Review Questions

1. Prove that  $\neg(\neg p \Leftrightarrow (r \vee p))$  is logically equivalent to  $r \Rightarrow p$ .
2. Use a proof by contradiction to show that if  $n$  is an integer and  $n^2$  is even, then  $n$  is even.
3. Is the following statement true or false?  
 $\forall x \text{ in } \mathbb{R}, \exists y \text{ in } \mathbb{R} \text{ such that } y \geq 0 \wedge (y = x \vee y = -x)$ .  
 Explain why your answer is correct.
4. Is the following statement true or false?  
 For all sets  $A, B$  and  $C$ ,  $A - (B \cup C) \subseteq (A - B) \cap (A - C)$ .  
 Prove your answer is correct.
5. Let  $f : B \rightarrow C$  and  $g : A \rightarrow B$ . Prove that if  $f \circ g$  is onto then  $f$  is onto.
6. Prove that  $x^3 - 10x^2$  is *not*  $O(x^2)$ .
7. Prove that for every positive integer  $n$ ,  $\sum_{k=1}^n k2^k = (n-1)2^{n+1} + 2$ .
8. Yark University has 40000 students. Each student takes 5 classes each term. The University offers 1000 classes each term. The largest classroom at Yark holds 180 students. Is this a problem? Explain why.
9. Consider the domain of all people.  
 Let  $P(x, y)$  represent the statement “ $x$  is the parent of  $y$ ”.
  - (a) Translate the following formulas into clear and precise English.  
 $\exists x \forall y P(x, y)$   
 $\forall y \exists x P(x, y)$
  - (b) Express the statement “Somebody has no grandchildren” using only the predicate  $P$ .
10. Prove that for all integers  $a, b$  and  $c$ , the product of some pair of the three integers is non-negative. (In other words, show that  $ab \geq 0$  or  $ac \geq 0$  or  $bc \geq 0$ .)
11. Let  $A = \{0, 1, 2\}$  and  $B = \{1, 3\}$ . List the elements of each of the following sets.
  - (a)  $A - B =$
  - (b)  $A \times B =$
12. Let  $f : A \rightarrow B$ . Let  $S$  and  $T$  be subsets of  $A$ .
  - (a) Prove that  $f(S \cap T) \subseteq f(S) \cap f(T)$ .

(b) Give an example of a function  $f$  and sets  $S$  and  $T$  such that  $f(S \cap T) \neq f(S) \cap f(T)$ . Briefly explain why your answer is correct.

13. Give a good lower bound on  $\sum_{i=1}^n i^{1.5} (\log_2 i)^2$  using  $\Omega$  notation. Prove your answer is correct.

14. Use the integral method to get a good upper bound on  $\sum_{i=0}^n i e^{2i^2}$ .

15. Is  $\Omega(n) \subseteq \Omega(n + \sqrt{n})$ ? Prove your answer is correct.

16. Is  $3^{\log n} \in O(2^{\log n})$ ? Prove your answer is correct.

17. Beside each function  $f(n)$  in the left column, write down the number of the **first** function  $g(n)$  in the right column such that  $f(n) \in O(g(n))$ .

(a)  $n^{1.9} (\log_2 n)^2$  \_\_\_\_\_

1. 1

(b)  $\sum_{i=1}^n \sqrt{i} \log_2 i$  \_\_\_\_\_

2.  $\log n$

3.  $\sqrt{n}$

(c)  $(n\sqrt{n} + 7n + 2)^2$  \_\_\_\_\_

4.  $\sqrt{n} \log n$

(d)  $10 \cdot 2^{\log_4 n}$  \_\_\_\_\_

5.  $n$

(e)  $\sum_{i=1}^n \frac{n}{i \log n}$  \_\_\_\_\_

6.  $n \log n$

7.  $n^2$

8.  $n^3$

9.  $2^n$

10.  $n^n$

18. Prove  $n^2 \leq 2^n$  for all natural numbers  $n \geq 4$ .

19. Let  $f: A \rightarrow B$  be a function. For any set  $C \subseteq B$ , define  $f^{-1}(C)$  to be the set  $\{a \in A : f(a) \in C\}$ . Prove that for every  $f: A \rightarrow B$  and subsets  $S$  and  $T$  of  $B$  we have  $f^{-1}(S) \cap f^{-1}(T) = f^{-1}(S \cap T)$ .

20. Show that if 51 distinct numbers are chosen among  $\{1, 2, 3, \dots, 100\}$  then there must be two numbers among them whose sum is 101.