Homework Assignment #5
Due: October 15, 2015 at 4:00 p.m.

1. Suppose you are given two arrays $A[1..n]$ and $B[1..n]$ of integers. Each array is sorted in increasing order. The $2n$ elements stored in the two arrays are all distinct. The goal of this problem is to design an algorithm to select the kth smallest element among the $2n$ elements (where $1 \leq k \leq 2n$).

(a) An easy way to solve the problem is to merge the two arrays into a single sorted array of size $2n$, and then return the element in the kth position of the merged array. How many steps would this take in the worst case? (Use big-Θ notation to express your answer in terms of k and/or n.)

(b) Fill in the five blanks in the following divide-and-conquer algorithm. You must explain how you came up with your answers, but you need not give a formal proof that the algorithm is correct.

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SELECT(lo, hi, k)
    Preconditions: 1 ≤ lo ≤ hi + 1 ≤ n + 1, A and B are sorted and
            all elements of A and B are distinct, and 1 ≤ k ≤ n + hi − lo + 1
    if lo = hi + 1 then
        return ________________________
    else
        mid ← ⌈(lo+hi)/2⌉
        use a binary search of B[1..n] to find the number of elements m in
                B that are less than A[mid]
        if ________________________ then
            return A[mid]
        else if ________________________ then
            return SELECT(mid + 1, hi, ________________________)
        else
            return SELECT(lo, mid − 1, ________________________)
        end if
    end if
    Postcondition: returns the kth smallest element in
            {A[j] : lo ≤ j ≤ hi} ∪ {B[j] : 1 ≤ j ≤ n}
end SELECT
```

(c) Using big-Θ notation, state the worst-case running time of the algorithm in part (b) in terms of lo, hi, n and/or k. Explain why your answer is correct.

(d) To solve the original problem, you would call SELECT$(1, n, k)$. Using big-Θ notation, state the worst-case running time of this call in terms of n and/or k. Explain why your answer is correct.